Circles, Angle Measures and Arcs

A.5C	Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions
A.6G	Relate direct variation to linear functions and solve problems involving proportional change.
A.7A	Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;
G.2A	Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.
G.2B	Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
G.3D	Use inductive reasoning to formulate a conjecture.
G.9C	Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

Materials

Advance Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use Geometer's Sketchpad as a class demonstration tool.

For each student:

- Graphing calculator
- **Create an "Arc Measuring Tool"** activity sheet
- **Angles Formed by Chords Intersecting Inside a Circle** activity sheet
- **Angles Formed by Secants Intersecting Outside a Circle** activity sheet
- **Other Intersecting Lines and Segments** activity sheet
- **Quad-Tri Incorporated** activity sheet

For each student group of 3 - 4 students:

- Compasses
- Protractors
- Patty paper or tracing paper
- Rulers
- Scissors

ENGAGE

The Engage portion of the lesson is designed to create student interest in the relationships among the measures of angles formed by segments in circles and related arc measures. This part of the lesson is designed for groups of three to four students.

1. Distribute two sheets of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on one sheet of patty paper. Then have them construct a second circle, congruent to the first circle on the second sheet of patty paper.
3. Distribute the **Create an Arc Measuring Tool** activity sheet. Students should follow the directions on the sheet.
4. On their second circle, students should draw two intersecting chords that do not intersect in the center of the circle.
5. Students should use the available measuring tools to find angle measures and estimate arc measures.
6. Students will record their individual results, share results with their group, and discuss observations.
7. Debrief the activity using the Facilitation Questions.

Facilitation Questions – Engage Phase

1. When you fold a diameter, how many degrees are in each semi-circle?
   180° semi means half; one-half of 360° is 180°.
2. When you fold a second diameter perpendicular to the first, how many degrees are in each quarter-circle?
   90° one quarter means one-fourth, one-fourth of 360° is 90°.
3. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*
4. How did you use your "Arc Measuring Tool" to estimate the measures of the arcs in your circle? *Answers may vary. Students should be able to explain how they used known "benchmarks" like 90°.*
5. What other method could you use to determine the measures of the arcs on your second circle?
   Answers may vary. Students should realize they can draw central angles that intercept the arc they are trying to measure and the measure of the central angle is equal to the measure of the intercepted arc.
6. What similarities do your measurements have with measurements taken by other members of your group?
   Answers may vary. Students may notice, vertical angles are congruent; the sum of the measures of all arcs of the circle is 360° etc.
7. How can you determine if your observations will be true for any circle?
   Answers may vary. Students should realize that data for several circles could be collected and analyzed to verify conjectures.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to participate actively in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet.
2. Students should open the sketch **Twochords-in.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
   Note: If students are not familiar with the operation of Geometer's Sketchpad, they will need the necessary instruction at this time.

Facilitation Questions – Explore Phase

1. What patterns do you notice in the table?
   Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles, etc.

2. Where do you see proportional relationships in your table?
   Properties of proportional relationships can be explored at this time. Remind students of scale factors and constant of proportionality.

3. How did you use your table to develop an algebraic rule for this relationship?
   Answers may vary. Students may have used the process column, constant of proportionality, finite differences, etc.

EXPLAIN

The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.

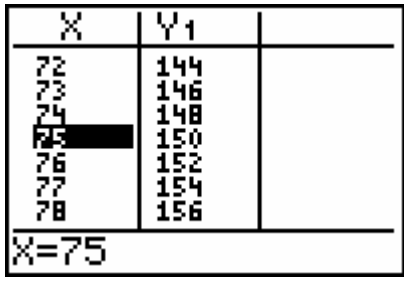
1. Debrief the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet. Use the Facilitation Questions to help students make connections among methods that can be used to calculate the measure of the angle or intercepted arc.
2. Have each student group present the way they found the algebraic rule and give a verbal description of the relationship.
3. Be sure students understand how to use the Geometer's Sketchpad sketches.

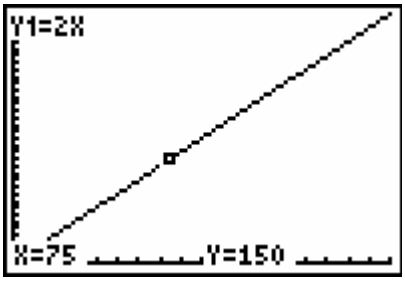
Facilitation Questions – Explain Phase

1. What is the meaning of your algebraic rule in this relationship?
   Two times the angle measure equals the sum of the intercepted arcs.
2. If you know the measure of the angle, how can you find the sum of the measures of the intercepted arcs?
   Multiply the angle measure by 2.
3. If you know the measure of each intercepted arc, how can you find the angle measure?
   Find the sum of the arcs and then divide by 2.
4. If you know the measure of one angle and one intercepted arc, how could you find the measure of the other intercepted arc?
   Double the angle measure then subtract the known arc from that value.
5. If you know the measure of one angle and one intercepted arc, what algebraic equation could you write to calculate the measure of the other intercepted arc?

$$2(angle) = arc1 + arc2$$

6. How could you use the table or graph feature of your graphing calculator to determine the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?





ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
2. Students should open the sketch **Twosecants-out.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
4. Debrief the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
5. Distribute the **Other Intersecting Lines and Segments** activity sheet.
6. Prompt students to open the sketches as directed and explore the relationships.
7. Debrief the **Other Intersecting Lines and Segments** activity sheet.

Facilitation Questions – Elaborate Phase

1. What patterns do you notice in the table?
   Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles etc.
2. Where do you see proportional relationships in your table?
   Properties of proportional relationships can be explored at this time. Remind students of Scale factors and constant of proportionality.
3. How did you use your table to develop an algebraic rule for this relationship?
   Answers may vary. Students may have used the process column, constant of proportionality, finite differences etc.
   After completing the summary table for this activity, what general statements can you make about angles formed by lines and segments that intersect circles?

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute the Mathematics Chart.
2. Provide each student with the **Quad-Tri Incorporated** activity sheet.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

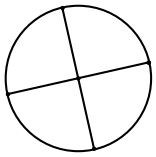
Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	G.9(c)	D	B	C	A		
2	G.9(c)	D	B	C	A		
3	G.9(c)	A	C	D	B		
4	G.9(c)	A	C	B	D		

Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

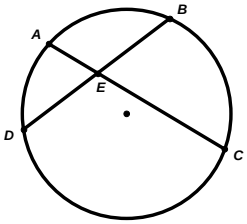
3. Fold a diameter in the second circle. Unfold the circle then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?
   The point is the center of the circle. It is the midpoint of the diameters so it must be the center.

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



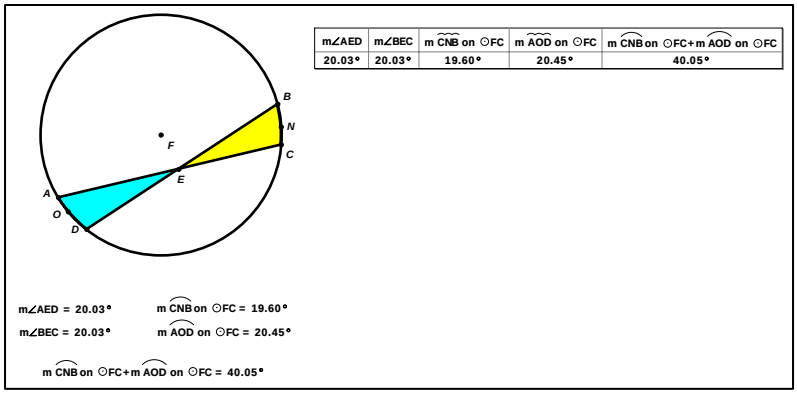
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$
	50°	*50°*	*60°*	*40°*
	65°	*65°*	*70°*	*60°*
	43°	*43°*	*40°*	*46°*
	124°	*124°*	*82°*	*166°*
	130°	*130°*	*100°*	*160°*

8. What patterns do you observe in the table?
   Answers may vary. Students should observe that the $m\angle AED = m\angle BEC$ or that the sum of the measures of the arcs is twice the measure of each angle.

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. **Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?**
   The measures change.
2. **Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.**
3. **Double click again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.**
4. **Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.**
5. **Record the data from the computer in the table below.**

$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$
20.03	20.03	19.60	20.45	40.05
35.53	35.53	50.61	20.45	71.06
42.57	42.57	64.69	20.45	85.14
56.60	56.60	64.69	48.51	113.20
68.98	68.98	64.69	73.28	137.97
79.68	79.68	86.09	73.28	159.37
96.54	96.54	119.79	73.28	193.07
125.02	125.02	119.79	130.24	250.03
144.07	144.07	119.79	168.35	288.14
170.00	170.00	171.65	168.35	340.00

6. What patterns do you observe in the table?

Answers may vary. Students should observe the $m\angle AED = m\angle BEC$ and the sum of the measures of the arcs is twice the measure of each angle.

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

$m\angle AED$ (x)	PROCESS	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ (y)
20.03	*(2) 20.03*	*40.05*
35.53	*(2) 35.53*	*71.06*
42.57	*(2) 42.57*	*85.14*
56.60	*(2) 56.60*	*113.20*
68.98	*(2) 68.98*	*137.97*
79.68	*(2) 79.68*	*159.37*
96.54	*(2) 96.54*	*193.07*
125.02	*(2) 125.02*	*250.03*
144.07	*(2) 144.07*	*288.14*
170.00	*(2) 170.00*	*340.00*
x	$2x$	y

8. Use the process column to develop an algebraic rule that describes this relationship.

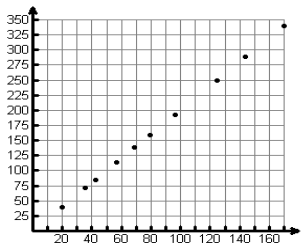
$y = 2x$

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

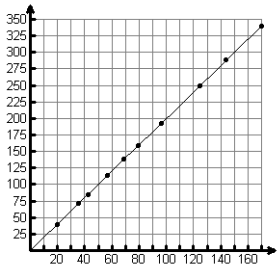
Two times the measure of the angle is equal to the sum of the measures of the intercepted arcs. The sum of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.

10. Create a scatterplot of the sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

x-min = *0*
x-max = *170*
y-min = *0*
y-max = 350



11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.



12. Does the graph verify your function rule? Why or why not?
Yes. The graph of the function rule passes through each data point.

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?
75°

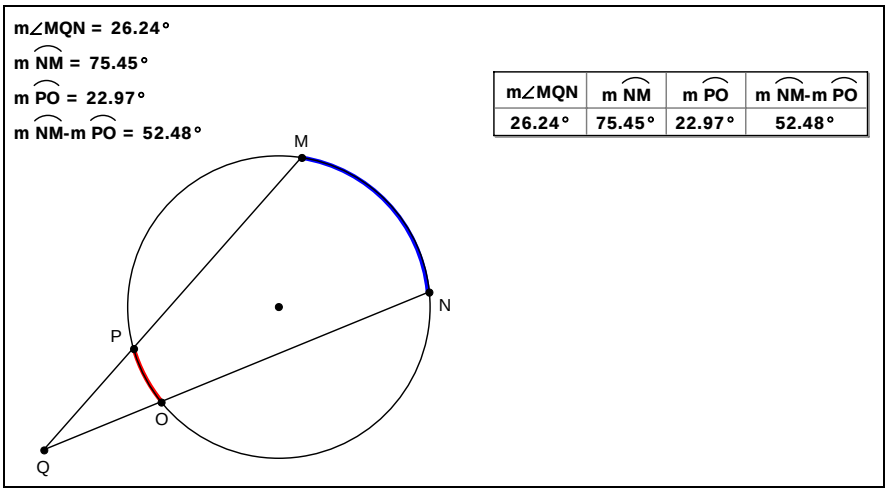
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?
112°

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.
To determine the measure of the angle, add the two intercepted arcs then divide by 2.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.
To determine the sum of the measures of the intercepted arcs, multiply the measure of the angle by 2

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecants-out**.



1. **Double click on the table to add another row, then click and drag point *M*. What do you observe?**
   The measures change.
2. **Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.**

3. **You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.**

4. **Record the data from the computer in the table below.**

$m\angle MQN$	$m\overset{\frown}{MN}$	$m\overset{\frown}{PO}$	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$
26.24	75.45	22.97	52.48
29.84	85.92	26.24	59.68
35.90	99.89	28.09	71.80
40.58	113.21	32.05	81.16
46.22	130.52	38.09	92.43
50.68	143.71	42.35	101.36
55.99	163.39	51.40	111.99
58.91	172.42	54.60	117.82
64.63	192.27	63.01	129.25
73.05	241.94	95.84	146.10

5. **What patterns do you observe in the table?**
Answers may vary. Students should observe the measure of the angle is one-half the difference of the measures of the intercepted arcs.

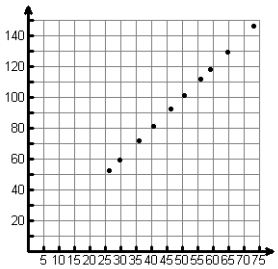
6. **To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.**

$m\angle MQN$ (x)	PROCESS	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$ (y)
26.24	*(2) 26.24*	*52.48*
29.84	*(2) 29.84*	*59.68*
35.90	*(2) 35.90*	*71.80*
40.58	*(2) 40.58*	*81.16*
46.22	*(2) 46.22*	*92.43*
50.68	*(2) 50.68*	*101.36*
55.99	*(2) 55.99*	*111.99*
58.91	*(2) 58.91*	*117.82*
64.63	*(2) 64.63*	*129.25*
73.05	*(2) 73.05*	*146.10*
x	$2x$	y

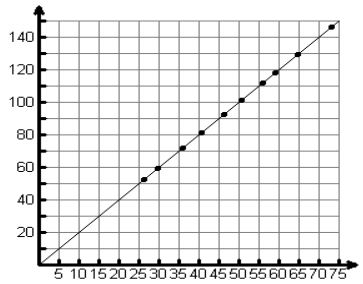
7. **Use the process column to develop an algebraic rule that describes this relationship.**
$y = 2x$

8. **Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.**
Two times the measure of the angle is equal to the difference of the measures of the intercepted arcs. The difference of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.

9. **Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window.**
x-min $= 0$
x-max $= 75$
y-min $= 0$
y-max $= 150$



10. **Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.**



11. **Does the graph verify your function rule? Why or why not?**
Yes. The graph of the function rule passes through each data point.

12. **What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?**
45°

13. **What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants 43°?**
86°

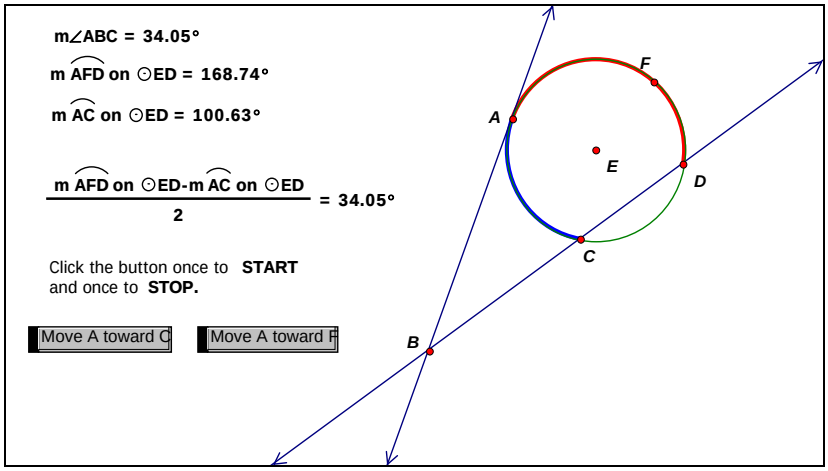
14. **Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.**
To determine the measure of the angle, subtract the measures of the two intercepted arcs then divide by 2.

15. **Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.**
To determine the difference of the measures of the intercepted arcs, multiply the measure of the angle by 2.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

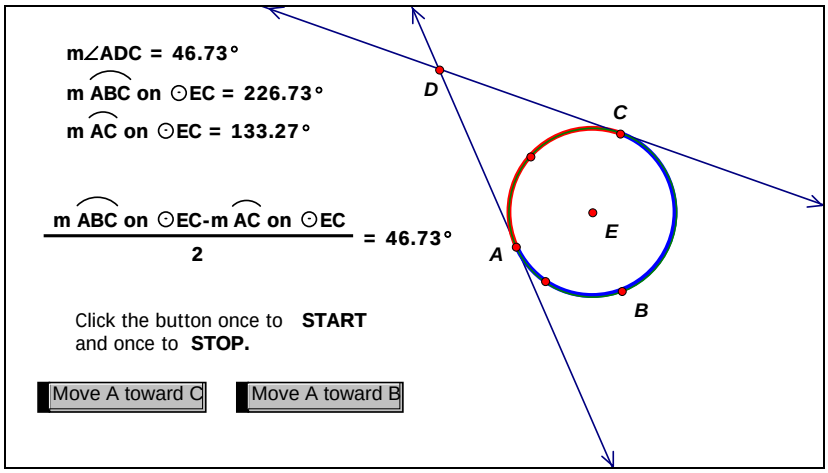
a. Open the sketch, "Tansecant-out.".



b. Click a button to move point A. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the difference in the measures of the intercepted arcs.

2. Two tangents that intersect in the exterior of a circle

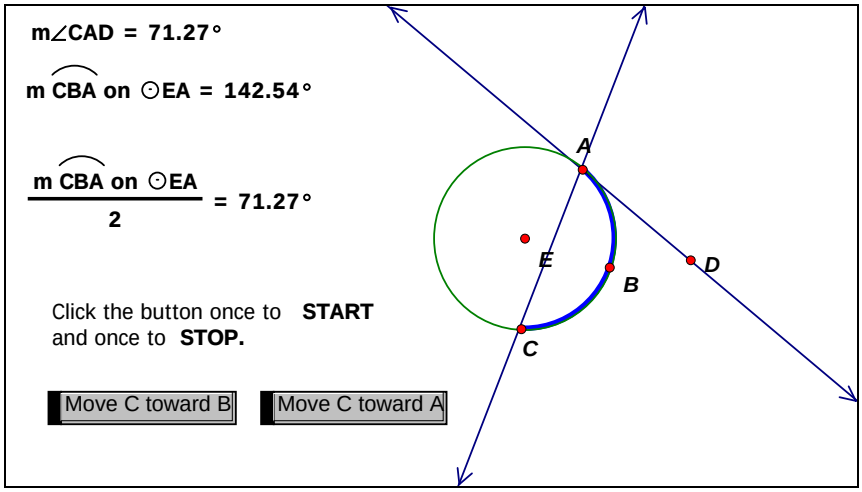
a. Open the sketch, "Twotangents-out."



b. Click a button to move point A. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the difference in the measures of the intercepted arcs.

3. Tangent and a Secant that intersect on a circle

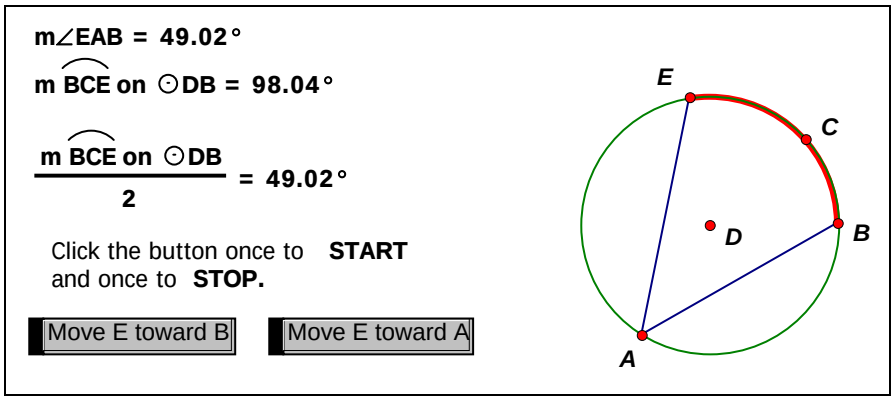
a. Open the sketch "Tansecant-on."



b. Click a button to move point C. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the measure of the intercepted arc.

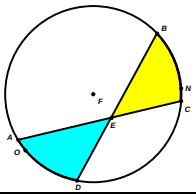
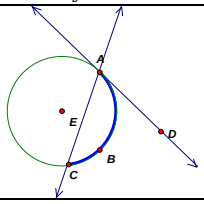
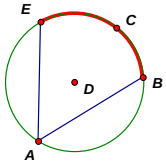
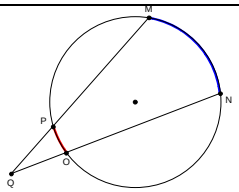
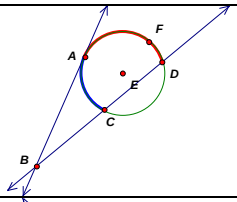
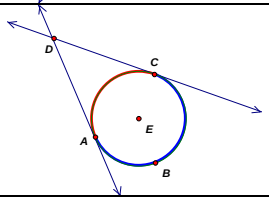
4. Two chords that intersect on a circle

a. Open the sketch "Twochords-on."



b. Click a button to move point *E*. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the measure of the intercepted arc.

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

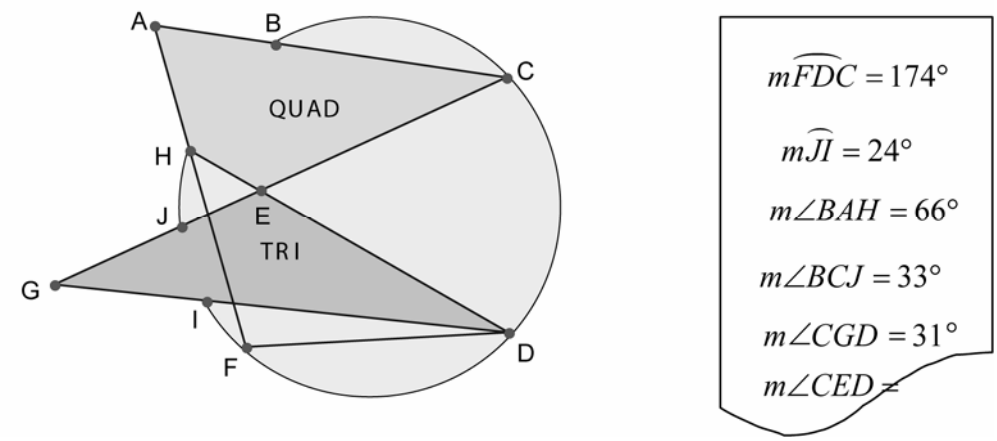
Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
	*Inside the circle*	*The measure of the angle is one-half the sum of the measures of the intercepted arcs.*
	*On the circle*	*The measure of the angle is one-half the measure of the intercepted arc.*
	*On the circle*	
	*Outside the circle*	*The measure of the angle is one-half the difference in the measures of the intercepted arcs.*
	*Outside the circle*	
	*Outside the circle*	

Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, _*add*_ the measures of the intercepted arcs then _*divide by 2*_.
2. When the vertex is **outside** the circle, *subtract the* measures of the intercepted arcs then _*divide by 2*_.
3. When the vertex is **on** the circle, *divide the measure of the intercepted arc by 2* .

Quad-Tri Incorporated

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.



Pierre thinks the measure of angle CED must be 60°. Is he correct? Justify your answer.

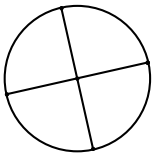
Answer: Pierre is not correct. Based on the known information, the measure of angle CED must be 55°.

Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

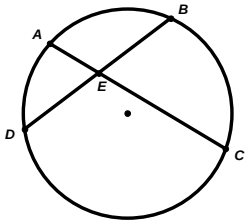
3. Fold a diameter in the second circle. Unfold the circle, then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool?

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



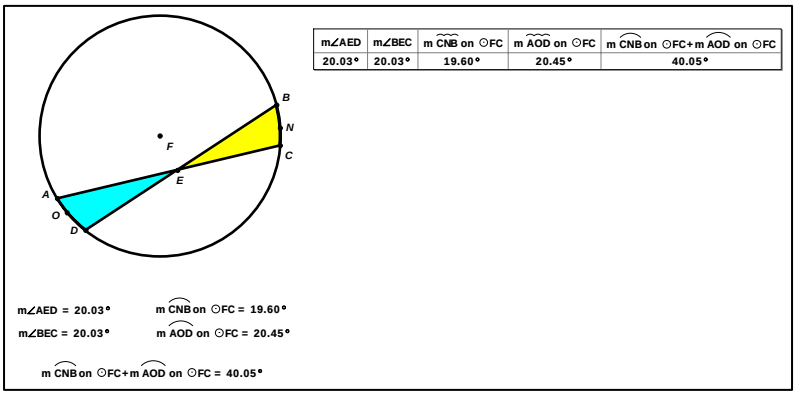
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$

8. What patterns do you observe in the table?

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. Double click on the table to add another row, then click and drag point *B* away from point *N.* What do you observe?

2. Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.

3. Double click on the table again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.

4. Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.

5. Record the data from the computer in the table below.

$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$

6. What patterns do you observe in the table?

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

$m\angle AED$ (x)	PROCESS	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ (y)
x		y

8. Use the process column to develop an algebraic rule that describes this relationship.

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

10. Create a scatterplot of sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

12. Does the graph verify your function rule? Why or why not?

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?

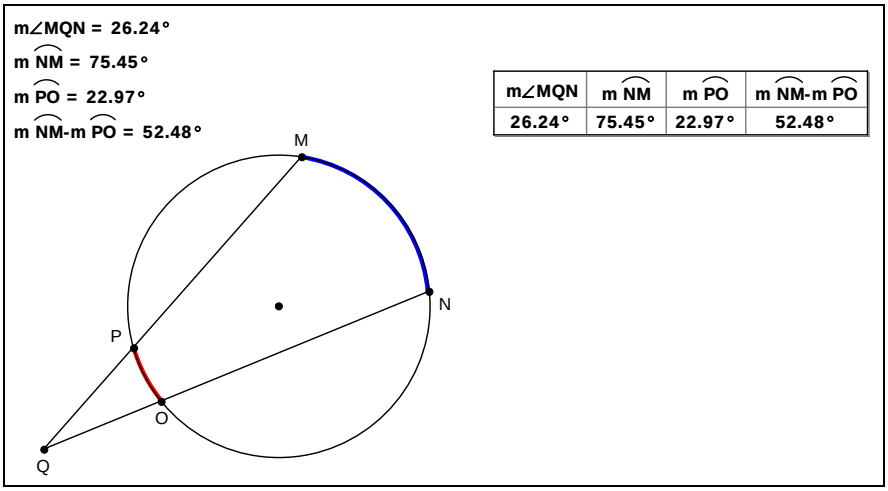
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecant-out**.



1. Double click on the table to add another row, then click and drag point *M*. What do you observe?

2. Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.

3. You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.

4. Record the data from the computer in the table below.

$m\angle MQN$	$m\overset{\frown}{MN}$	$m\overset{\frown}{PO}$	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$

5. What patterns do you observe in the table?

6. To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.

$m\angle MQN$ (x)	PROCESS	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$ (y)
x		y

7. Use the process column to develop an algebraic rule that describes this relationship.

8. Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.

9. Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window

   x-min =
   x-max =
   y-min =
   y-max =

10. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

11. Does the graph verify your function rule? Why or why not?

12. What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?

13. What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants is 43°?

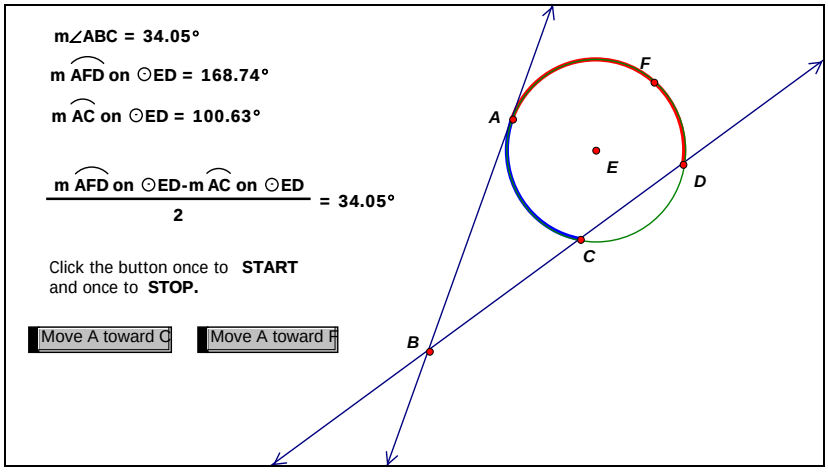
14. Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.

15. Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

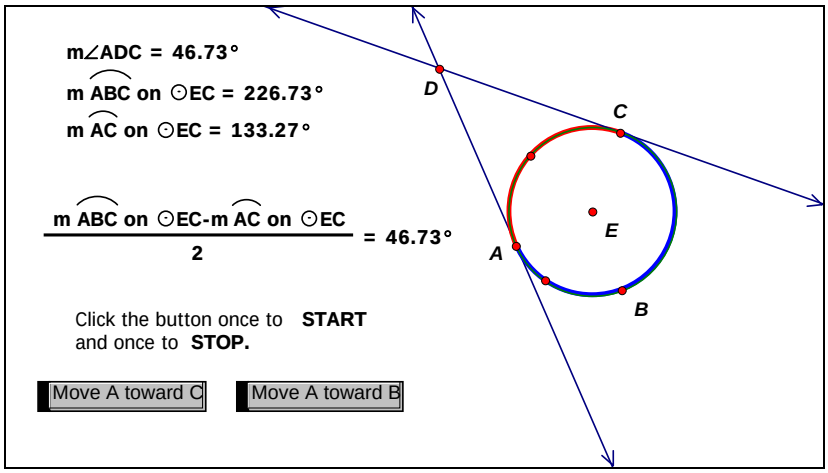
   a. Open the sketch, "**Tansecant-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

2. Two tangents that intersect in the exterior of a circle

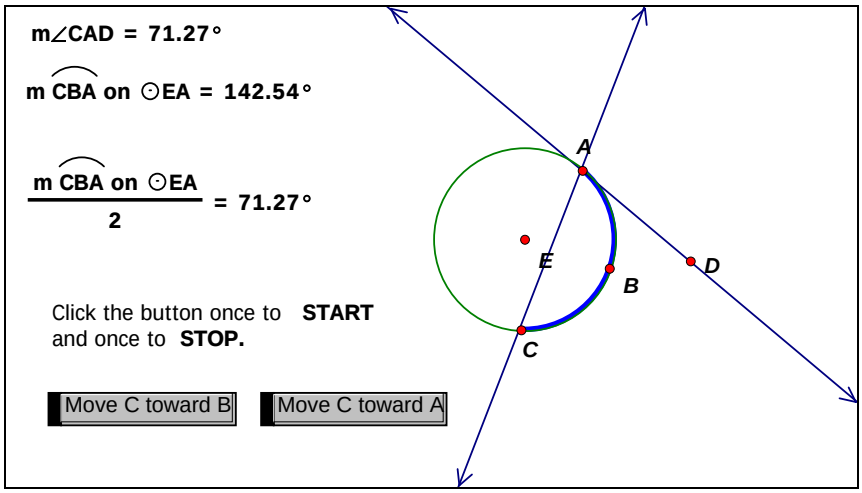
   a. Open the sketch, "**Twotangents-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

3. Tangent and a Secant that intersect on a circle

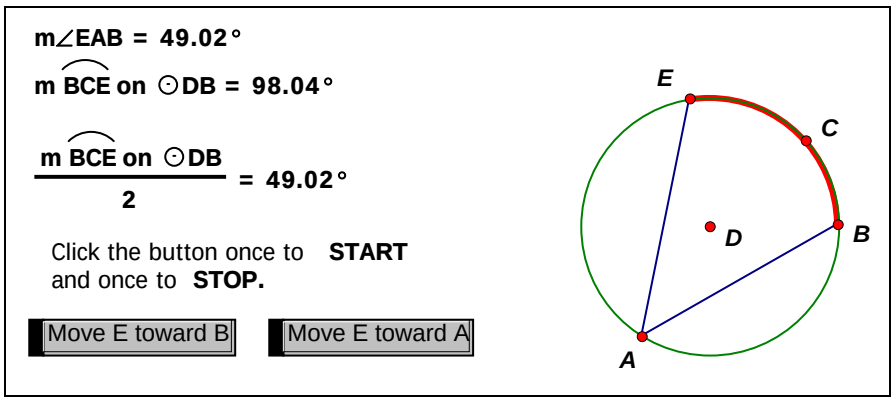
   a. Open the sketch "**Tansecant-on.**"



   b. Click a button to move point C. What do you observe about the angle and arc relationships?

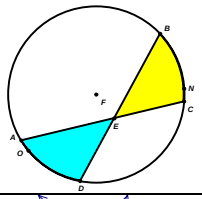
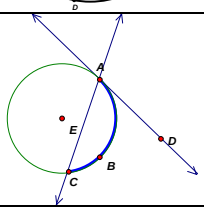
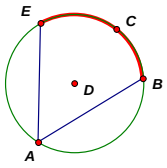
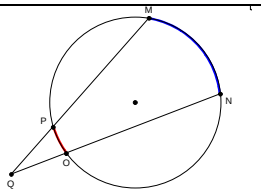
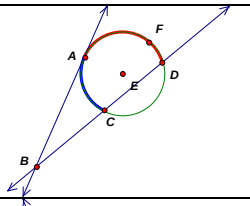
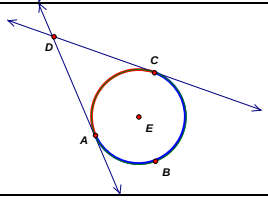
4. Two chords that intersect on a circle

   a. Open the sketch "**Twochords-on.**"



   b. Click a button to move point E. What do you observe about the angle and arc relationships?

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

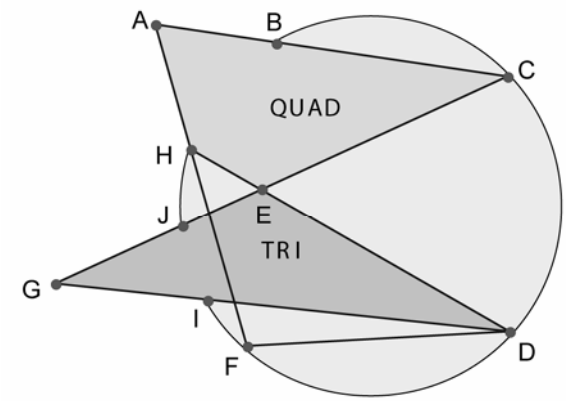
Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
		
		
		
		
		
		

Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, _____ the measures of the intercepted arcs then _____.
2. When the vertex is **outside** the circle, _____ the measures of the intercepted arcs then _____.
3. When the vertex is **on** the circle,_____.

Quad-Tri Incorporated

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.



$m\overset{\frown}{FDC} = 174°$

$m\overset{\frown}{JI} = 24°$

$m\angle BAH = 66°$

$m\angle BCJ = 33°$

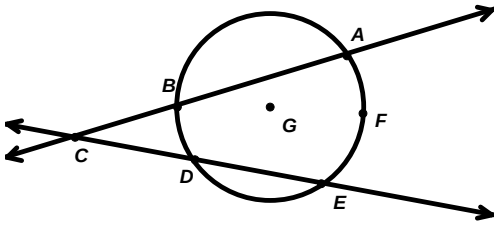
$m\angle CGD = 31°$

$m\angle CED =$

Pierre thinks the measure of angle *CED* must be 60°. Is he correct? Justify your answer.

Circles, Angle Measures and Arcs

1 In the diagram $m\angle BCD = 25°$ and $m\overset{\frown}{BD} = 33°$.



Find $m\overset{\frown}{AFE}$.

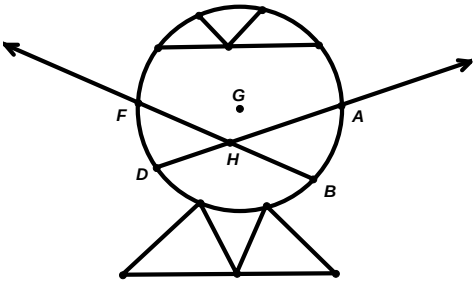
A 17°

B 50°

C 58°

D 83°

2 The metal sculpture shown was found in a recent archeological dig. $m\overset{\frown}{AB} = 46°$ and $m\overset{\frown}{FD} = 38°$



What is $m\angle DHB$?

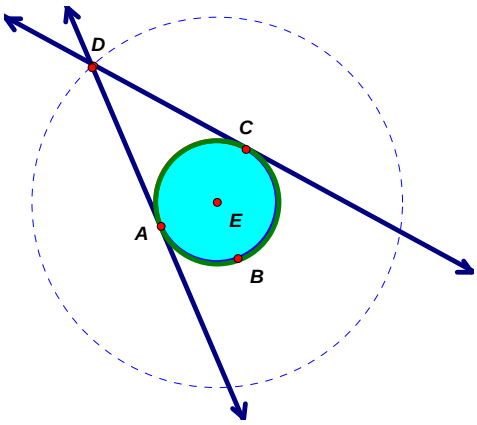
A 4°

B 42°

C 84°

D 138°

3 In the diagram, Point *D* represents a spacecraft as it orbits the Earth.



At this location 220° of the Earths surface is not visible from the spacecraft. What must be the $m\angle ADC$?

A 40°

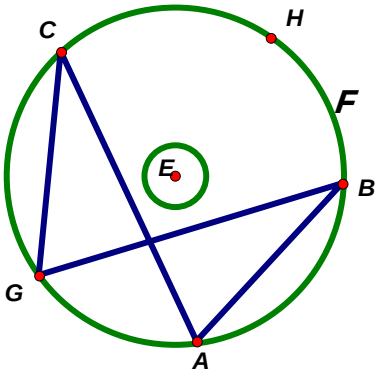
B 80°

C 110°

D 140°

4 Pablo created the sketch below.

m $\overset{\frown}{AB}$ on ⊙EF = 80°
m $\overset{\frown}{CG}$ on ⊙EF = 84°
m∠GBA = 31°



Based on the measurements he took, what must be $m\overset{\frown}{CHB}$?

A 134°

B 82°

C 67°

D 33.5°