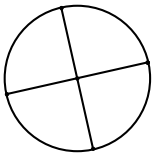
Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

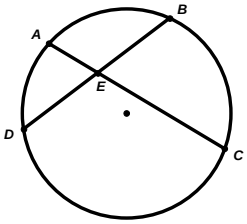
3. Fold a diameter in the second circle. Unfold the circle, then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool?

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



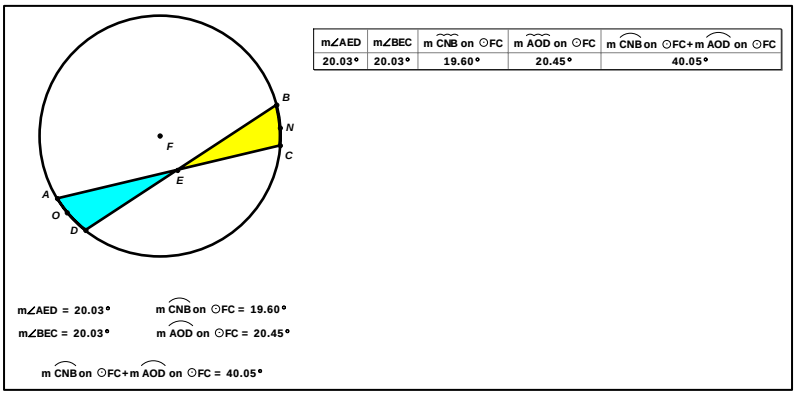
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$

8. What patterns do you observe in the table?

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. Double click on the table to add another row, then click and drag point *B* away from point *N.* What do you observe?

2. Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.

3. Double click on the table again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.

4. Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.

5. Record the data from the computer in the table below.

$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$

6. What patterns do you observe in the table?

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

$m\angle AED$ (x)	PROCESS	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ (y)
x		y

8. Use the process column to develop an algebraic rule that describes this relationship.

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

10. Create a scatterplot of sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

12. Does the graph verify your function rule? Why or why not?

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?

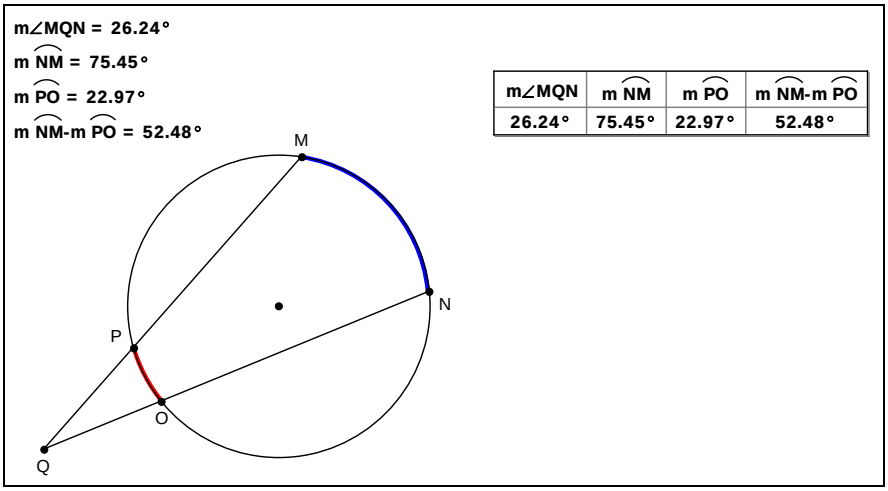
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecant-out**.



1. Double click on the table to add another row, then click and drag point *M*. What do you observe?

2. Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.

3. You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.

4. Record the data from the computer in the table below.

$m\angle MQN$	$m\overset{\frown}{MN}$	$m\overset{\frown}{PO}$	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$

5. What patterns do you observe in the table?

6. To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.

$m\angle MQN$ (x)	PROCESS	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$ (y)
x		y

7. Use the process column to develop an algebraic rule that describes this relationship.

8. Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.

9. Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window

   x-min =
   x-max =
   y-min =
   y-max =

10. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

11. Does the graph verify your function rule? Why or why not?

12. What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?

13. What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants is 43°?

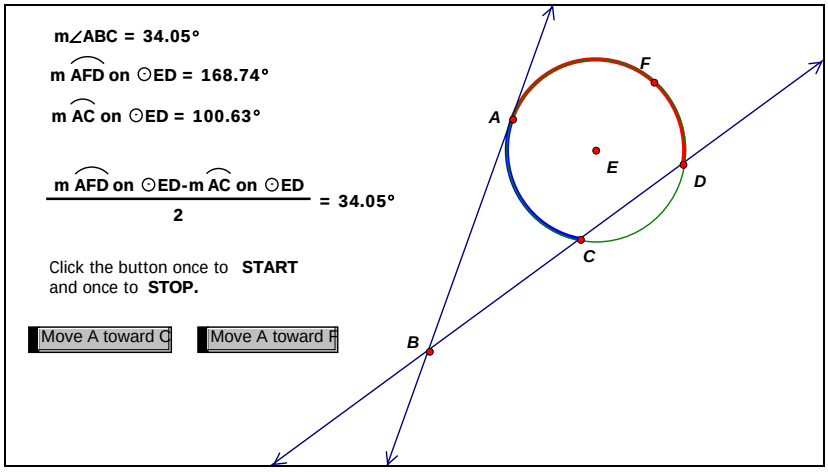
14. Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.

15. Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

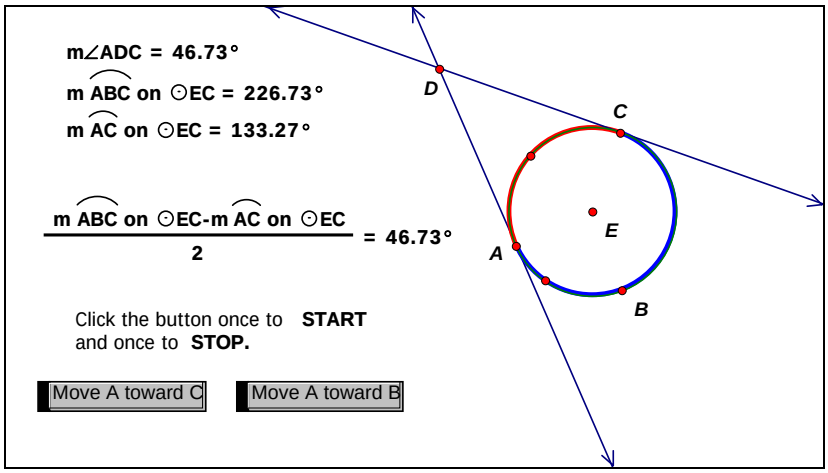
   a. Open the sketch, "**Tansecant-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

2. Two tangents that intersect in the exterior of a circle

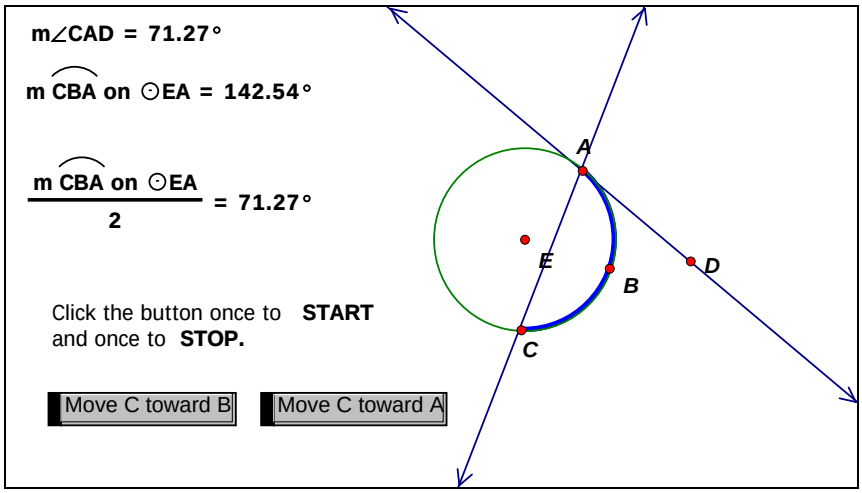
   a. Open the sketch, "**Twotangents-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

3. Tangent and a Secant that intersect on a circle

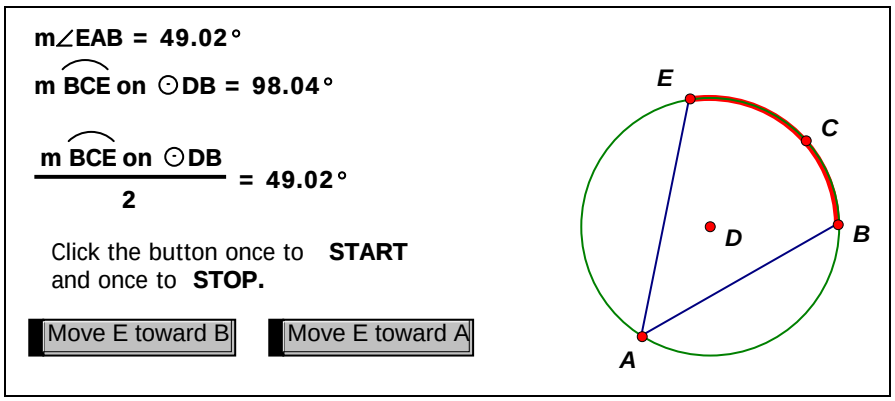
   a. Open the sketch "**Tansecant-on.**"



   b. Click a button to move point C. What do you observe about the angle and arc relationships?

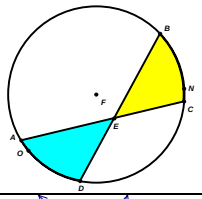
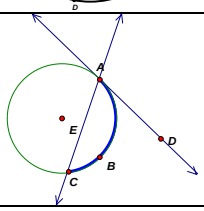
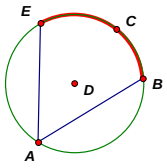
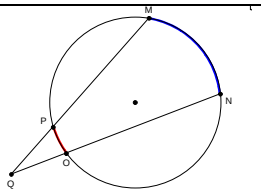
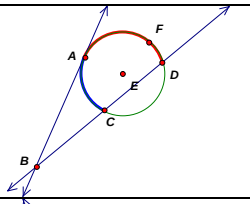
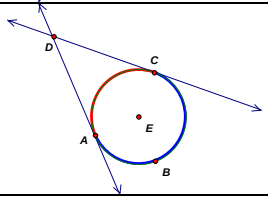
4. Two chords that intersect on a circle

   a. Open the sketch "**Twochords-on.**"



   b. Click a button to move point E. What do you observe about the angle and arc relationships?

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

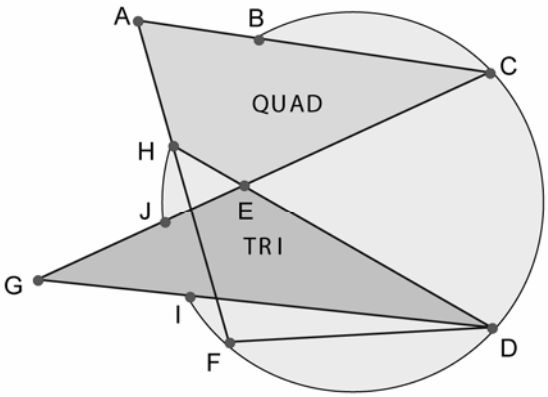
Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
		
		
		
		
		
		

Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, _____ the measures of the intercepted arcs then _____.
2. When the vertex is **outside** the circle, _____ the measures of the intercepted arcs then _____.
3. When the vertex is **on** the circle,_____.

Quad-Tri Incorporated

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.



$m\overset{\frown}{FDC} = 174°$

$m\overset{\frown}{JI} = 24°$

$m\angle BAH = 66°$

$m\angle BCJ = 33°$

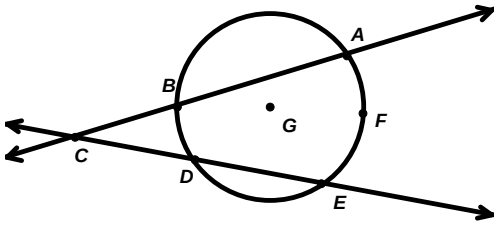
$m\angle CGD = 31°$

$m\angle CED =$

Pierre thinks the measure of angle *CED* must be 60°. Is he correct? Justify your answer.

Circles, Angle Measures and Arcs

1 In the diagram $m\angle BCD = 25°$ and $m\overset{\frown}{BD} = 33°$.



Find $m\overset{\frown}{AFE}$.

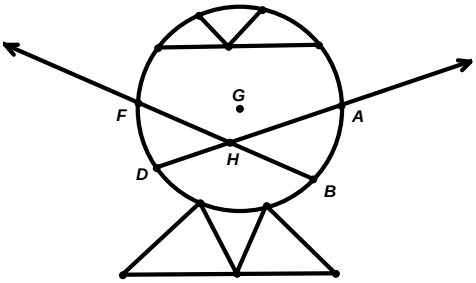
A 17°

B 50°

C 58°

D 83°

2 The metal sculpture shown was found in a recent archeological dig. $m\overset{\frown}{AB} = 46°$ and $m\overset{\frown}{FD} = 38°$



What is $m\angle DHB$?

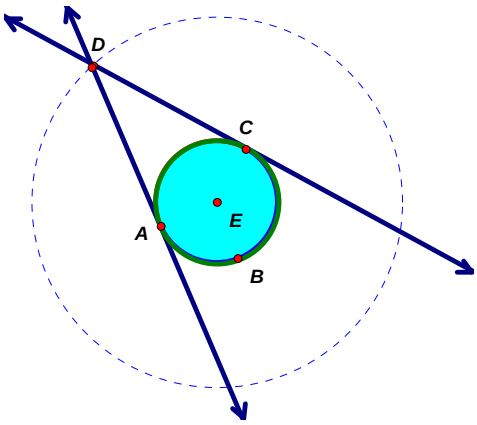
A 4°

B 42°

C 84°

D 138°

3 In the diagram, Point *D* represents a spacecraft as it orbits the Earth.



At this location 220° of the Earths surface is not visible from the spacecraft. What must be the $m\angle ADC$?

A 40°

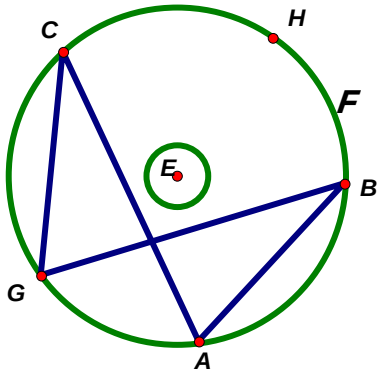
B 80°

C 110°

D 140°

4 Pablo created the sketch below.

m $\overset{\frown}{AB}$ on ⊙EF = 80°
m $\overset{\frown}{CG}$ on ⊙EF = 84°
m∠GBA = 31°



Based on the measurements he took, what must be $m\overset{\frown}{CHB}$?

A 134°

B 82°

C 67°

D 33.5°