Dome Floor Dilemma

Explore/Explain Cycle III

Purpose:

Provide participants the opportunity to use technology to explore relationships in geometric figures that yield quadratic data, such as change in area of a circle as the length of the radius changes. Participants will make connections between algebraic and geometric concepts that enhance their student's conceptual understanding of the Geometry TEKS.

Descriptor:

In a guided exploration, participants will create a sketch using Geometer's Sketchpad. They will collect and analyze data collected from their sketch using a variety of technologies. They will use problem-solving strategies of breaking a large problem into smaller components and working backwards to facilitate the constructions and the development of geometry concepts.

Duration:

2 hours

TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.8A Find areas of regular polygons, circles, and composite figures.

G.8B Find areas of sectors and arc lengths of circles using proportional reasoning.

G.9C Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

G.11D Describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

TAKS Objectives:

- Objective 3: Linear Functions
- Objective 4: Formulate and Use Linear Equations and Inequalities
- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of Geometric Relationships and Shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

Technology:

- Spreadsheet technology
- Hand-held graphing calculator
- Dynamic geometry software (Geometer's Sketchpad)
- Graph link technology

Materials:

Advance Preparation:

- Participant access to computers with Geometer's Sketchpad(latest version update available from http://www.keypress.com/sketchpad) and necessary sketches and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool
- Sketch **arcsegment.gsp** found on the CD (for leader's information).

For each group of two:

- Computer
- Copy of the **Technology Tutorial T²**

For each participant:

- Dome Floor Dilemma activity sheets
- Analyze the Data activity sheets
- Explain activity sheet
- Dome Floor Dilemma Intentional Use of Data (printed on green paper)

Leader Notes:

In this exploration participants will use Geometer's Sketchpad to create a sketch. They will use the sketch to collect and analyze data to discover the relationship between the length of the radius of a circle and the area of a sector and segment with a 60° arc. Specific details for using Geometer's Sketchpad are found in the ***Technology Tutorial T^2--Dome Floor Dilemma.***

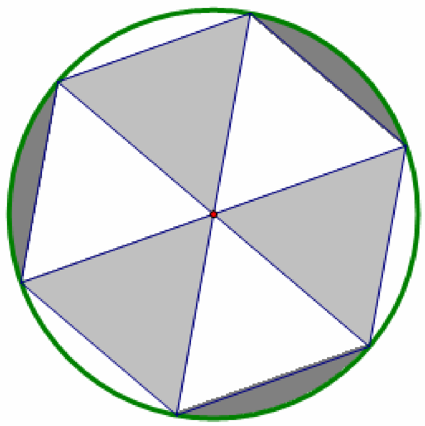
Participants will gather the data and analyze it on their own using their choice of the Geometer's Sketchpad, graphing calculator, TI-Interactive, spreadsheet, etc. During the discussion of the Explain phase, they will discuss several methods of analyzing the data and identify comparative advantages and disadvantages of each method.

Dome Floor Dilemma

Explore

Posing the Problem:

The diagram below represents the tile pattern on the circular floor of a domed building. Each shade, light, medium, and dark, represents a different color of floor tile. Each central angle is congruent to all others.



If you know the length of the radius of the circular floor, is it possible to calculate the area of each shaded region?

To answer this question participants will complete the explore activity.

Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, you will construct a geometric figure. then collect and analyze data. You will determine three functional relationships: area of a sector of a circle versus the radius, area of a segment of a circle versus the radius, and the area of the triangle bound by the segment and the radii drawn to the endpoints of the arc of the segment.

The Sector Construction

For detailed instructions on Geometer's Sketchpad see the Technology Tutorial T^2—Dome Floor Dilemma.

1. **Construct a circle with a radius.**

2. **Rotate the radius and the endpoint that lies on the circle 60^o.**

3. **Construct the intercepted arc of the sector.**

4. **Construct the interior of the sector.**

5. **Measure the length of the radius and the area of the sector.**

6. **Create a table to compare the two measurements. Which one is the independent variable and which one is the dependent variable?**

   The independent variable is the length of the radius and the dependent variable is the area of the sector.

7. **Plot the two measurements on a graph and turn on the trace feature.**

Collect the Data

6. **Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row. then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?**

   The measures change. The points are plotted and traced to create a graph.

7. **Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.**

8. **What patterns do you observe in the table?**

   Participants may observe that there is not a constant rate of change.

9. **What observations can you make about your graph?**

   Participants may observe that the graph appears to be quadratic.

The Arc Segment Construction

1. Construct the arc segment.

2. Change the color of the segment.

3. Measure the area of the segment.

4. Create a table to compare the measure of the area of the arc segment and the length of the radius. Which one is the independent variable and which one is the dependent variable?

The independent variable is the length of the radius and the independent variable is the area of the arc segment.

5. Plot the two measurements on the graph and turn on the trace feature.

Participants might have trouble with the trace feature if they have extra items highlighted on their screen when they choose the trace button. Remind them often to click in the blank white space.

Collect the Data

6. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

Possible answers might include: The measures change. The points are plotted and traced.

7. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

8. What patterns do you observe in the table?

Participants may observe that there is not a constant rate of change.

9. What observations can you make about your graph?

Participants may observe that the graph appears to be quadratic.

The Triangle Construction

1. Construct the triangle interior.

2. Measure the area of the triangle.

3. Create a table to compare the area of the triangle to the length of the radius. Which one is the independent variable? Which one is the dependent variable?

The independent variable is the length of the radius and the dependent variable is the area.

4. Plot the two measurements on the graph and turn on the trace feature.

Collect the Data

5. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

The measures change. The points are plotted and traced.

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

Participants may observe that this is not a constant rate of change.

8. What observations can you make about your graph?

Participants may observe that the graph appears to be quadratic.

Analyze the Data

1. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the sector, *y*.**

   $y = \frac{\pi x^2}{6}$

2. **Verify that your function rule models your data. Explain your verification.**

   Participants may graph the function rule over the scatterplot or verify using a table.

3. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the triangle, *y*.**

   $y = \frac{x^2\sqrt{3}}{4}$

4. **Verify that your function rule models your data. Explain your verification.**

   Participants may graph the function rule over the scatterplot or verify using a table.

5. **Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the segment, *y*.**

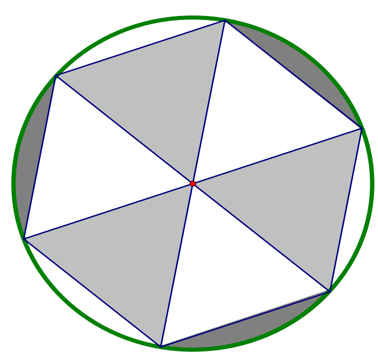
   $y = \frac{\pi x^2}{6} - \frac{x^2\sqrt{3}}{4}$

6. **Verify that your function rule models your data. Explain your verification.**

   Participants may graph the function rule over the scatterplot or verify using a table.

7. **Recall the floor design discussed earlier. The radius of the circle is 45 feet in length and the cost of tiling the different areas is listed below.**

   Un-shaded areas - $10.50 per square foot,
   Medium shaded areas - $12.00 per square foot and the
   Darkest shaded areas - $17.45 per square foot.



   Approximately what will be the total cost of tiling the floor?

   The total cost will be $75,394.30.

As a segue into the Explain Phase use the following facilitation questions to discuss the data collection and data analysis of the Dome Floor Dilemma.

Facilitation Questions
Data Collection:

- Why does the area not increase at the same rate as the radius?
  Area involves squaring the radius.
- What relationships are there among the 3 data sets?
  For equal radii the area of the sector is always greatest, followed by the triangle, then the segment.

Data Analysis:

- What type of function does this appear to be?
  Quadratic
- What is the parent function for this family?
  $y = x^2$
- How can you use geometric properties to determine the function rules?
  Answers may vary. Using the concept of composite area, segment area + triangle area = sector area.
- Why are the graphs of the functions similar?
  They are all area functions and of the family $y = x^2$.
- Why are the graphs of the functions different?
  They are increasing at different rates.

Explain

In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator and to some degree Geometer's Sketchpad. If none of the participant groups used a calculator, ask them how that method could have been used to analyze the data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.

1. **What knowledge of geometric properties was necessary to complete the constructions?**
   Answers may vary. Participants should discuss the properties of circles. For example the sum of the measures of the central angles of a circle is 360°.

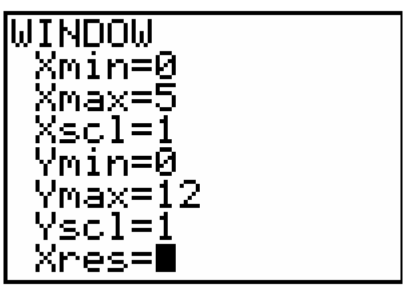
2. **What process did you use to develop your algebraic rules?**
Participants should share their methods. Sample methods appear below. If participants do not discuss each of these methods, the leader will bring them into the discussion.

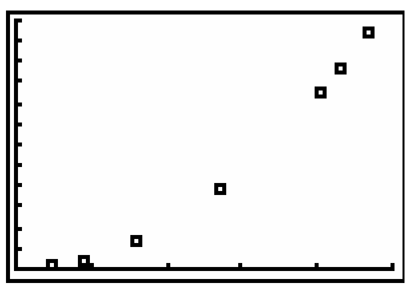
Area of a sector*: Since the area of the whole circle is $A = \pi r^2$ and in this case there are 6 sectors, the area of one sector is $A_{sec} = \frac{\pi r^2}{6}$. In this case the triangle is equilateral; therefore, its area can be expressed as $A = \frac{s^2\sqrt{3}}{4}$ where s represents the length of a side of the triangle. Since the triangle is equilateral s = r s,o the area can be expressed as $A = \frac{r^2\sqrt{3}}{4}$. To calculate the area of the segment, subtract the area of the triangle from the area of the sector. So the area of the segment is $A_{seg} = \frac{\pi r^2}{6} - \frac{r^2\sqrt{3}}{4}$.*

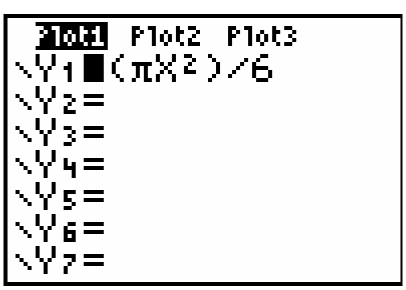
3. **How did you verify your function rules?**

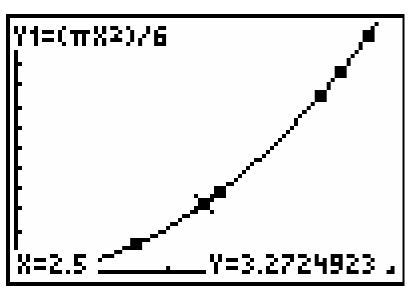
Participants may have created a scatterplot using a graphing calculato,r then graphed the rule over the scatter plot.

Sector

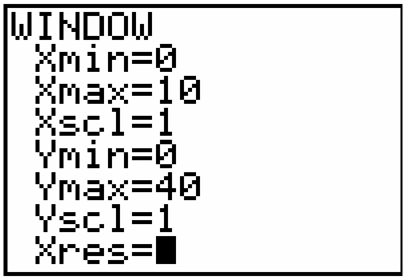


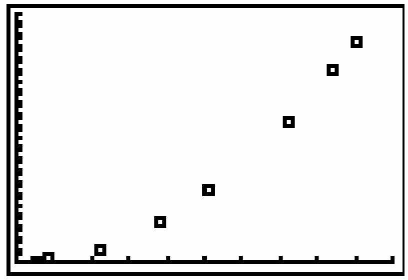


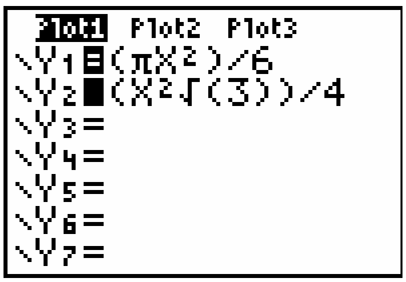


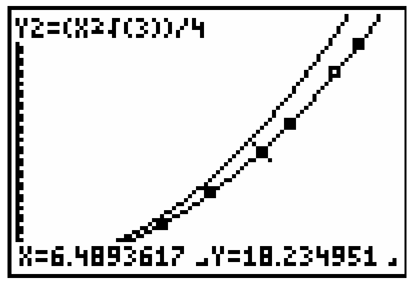


Triangle

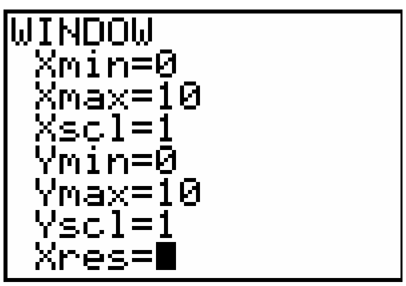


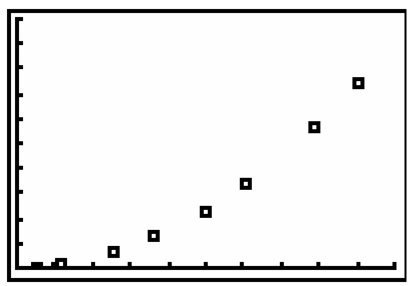


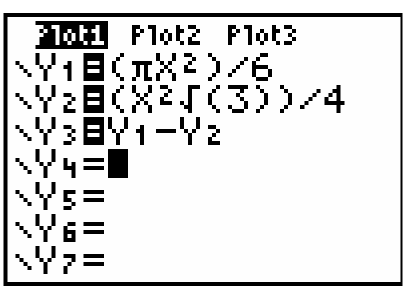


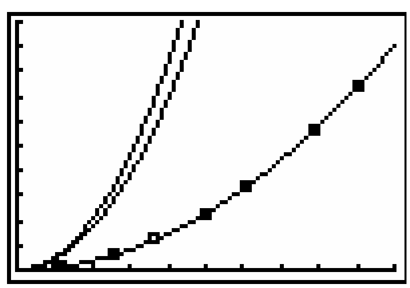


Segment









Participants may have verified the area of the segment symbolically and graphically.

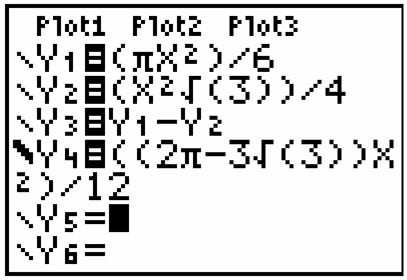
$$segment = sector - triangle$$

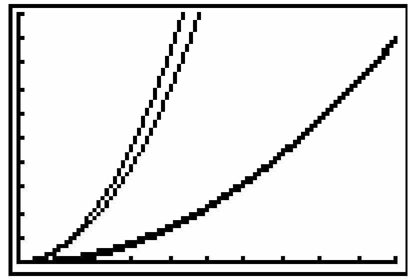
$$segment = \frac{\pi}{6}x^2 - \frac{\sqrt{3}}{4}x^2$$

$$segment = \frac{4\pi}{24}x^2 - \frac{6\sqrt{3}}{24}x^2$$

$$segment = \frac{4\pi - 6\sqrt{3}}{24}x^2$$

$$segment = \frac{2\pi - 3\sqrt{3}}{12}x^2$$



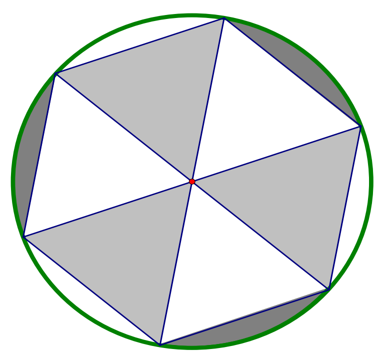


Explain how to verify the function rule using Geometer's Sketchpad. (See ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.****)*
Explain how to verify the function rule using TI-Interactive. (See ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.*** *)*
Explain how to verify the function rule using Spreadsheet. (See ***Technology Tutorial T² Dome Floor Dilemma—Function Rule Verification.****)*

4. **How did you solve the dome floor dilemma?**

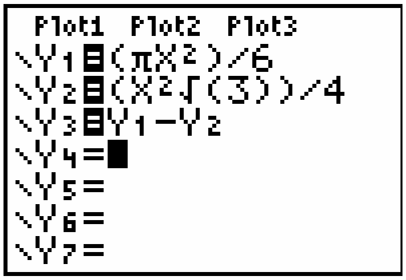
Recall the floor design discussed earlier. The radius of the circle is 45 feet in lengt, and the cost of tiling the different areas is listed below.

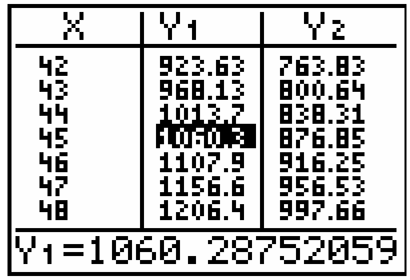
Un-shaded areas - \$10.50 per square foot,
Medium shaded areas - \$12.00 per square foot and the
Darkest shaded areas - \$17.45 per square foot.

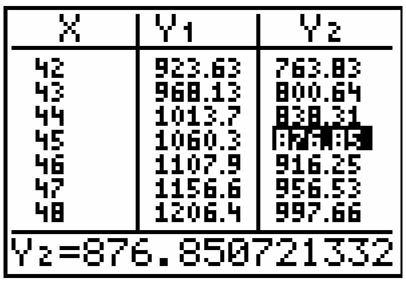


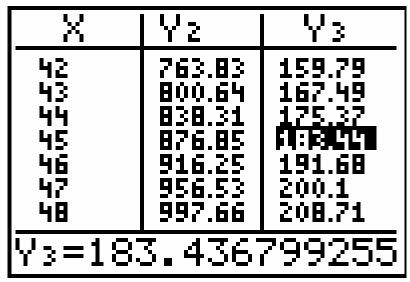
Approximately what will be the total cost of tiling the floor?

Participants may have used the table feature of the calculator to determine the areas of the different regions.









The area of one sector is approximately 1,060.29 square feet. Since there are 3 medium-shaded sectors, multiply 3 times 1,060.29. This gives 3,180.87 square feet at a cost of $12.00 per square foot, for a total of $38,170.44.

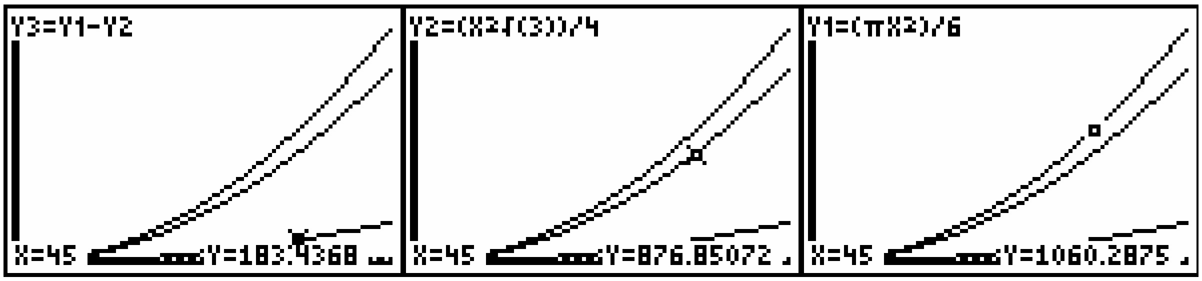
The area of one triangle is approximately 876.85 square feet. Since there are 3unshaded triangles, multiply 3 times 876.85. This gives 2,630.55 square feet at a cost of $10.50 per square foot, for a total of $27,620.78.

The area of one dark shaded segment is approximately 183.44 square feet. Since there are 3 segments, multiply 3 times 183.44. This gives 550.32 square feet at a cost of $17.45 per square foot, for a total of $9,603.08.

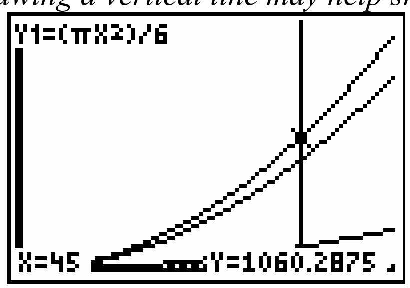
The total cost will be $75,394.30.

5. **How can you explain your graph of all three functions in a geometric context?**

The graphs are all quadratic because we are calculating area. Each is a transformation of the other because as the radius changes the area of each region grows at a different rate. At any value for the radius the sum of the y values for the segment and triangle functions will equal the y value for the sector function. This verifies that the area of a segment equals the area of the sector minus the area of the triangle.



Drawing a vertical line may help show this relationship.



Note to Leader: *Record or have a participant volunteer to record the responses to Questions 4, 5, and 6 on chart paper to use in the Elaborate phase of the professional development.*

6. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**

Responses may vary.

The data analysis takes only a few keystrokes. The power to set your own parameters and graph the function rule empowers the participant to use numerical analysis to calculate meaningful parameters such as a constant of variation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines. However, the small screen is difficult to see, and the axes in the window cannot be labeled.

7. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**

   Responses may vary.

   The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.

   However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

8. **What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?**

   Responses may vary.

   Like the graphing calculator, data analysis requires only a few keystrokes and clicks. The function editor enables participants to set their own rational function, empowering them to choose parameters that make physical sense in the context of the problem. The graphical analysis features of TI-Interactive make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

   Like the spreadsheet, axes can be labeled numerically and with text. The graphs are cleaner and can be copied and pasted into other computer documents.

9. **How will the use of these technologies promote a better understanding of the targeted mathematical concepts?**

   Answers will vary. The technology makes data collection and analysis a less time-consuming process, allowing more time to explore and connect the geometric concepts.

Dome Floor Dilemma
Intentional Use of Data—Leader Notes

1. *At the close of the Explain phase, distribute the* ***Dome Floor Dilemma Intentional Use of Data*** *activity sheet to each participant.*

2. *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to identify two key questions emphasized during this activity. Allow four minutes for discussion.*

> Facilitation Questions
> - Which TEKS formed the primary focus of this activity?
> - What additional TEKS supported the primary TEKS?
> - How do these TEKS translate into guiding questions to facilitate student exploration of the content?
> - How do your questions reflect the depth and complexity of the TEKS?
> - How do your questions support the use of technology?

3. *As a whole group, share responses for two to three minutes.*

4. *As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.*

> Facilitation Question
> - What attributes of the activity support the level of rigor that you identified?

5. *As a whole group, discuss how this activity might function in other settings. Allow five minutes for discussion.*

Facilitation Questions

- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

6. *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will provide prompts for generating attributes of judicious users of technology during the elaborate phase..*

Dome Floor Dilemma Intentional Use of Data

(possible participant answers)

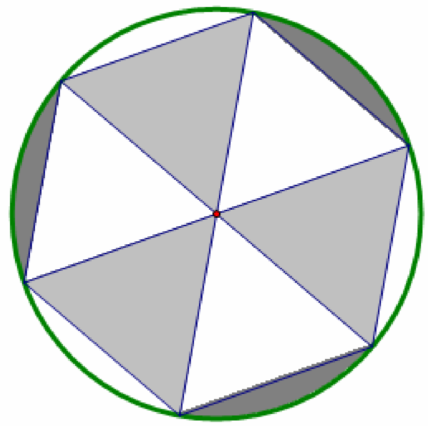
TEKS		*a(5), a(6), G.8A, B.8B, G.9C, G.11D*
Question(s) to Pose to Students	**Math**	*What type of relationships could be found among the measurements you gathered?*
	Tech	*How did technology help you solve the floor tile problem?*
Cognitive Rigor	**Knowledge**	√
	Understanding	√
	Application	√
	Analysis	√
	Evaluation	√
	Creation	√
Data Source(s)	**Real-Time**	*When using the computer sketch.*
	Archival	*none*
	Categorical	*none*
	Numerical	*none*
Setting	**Computer Lab**	*Each student uses the computer.*
	Mini-Lab	*In groups students take turns or groups switch out.*
	One Computer	*A student operates the control as other students read directions, entire class records data.*
	Graphing Calculator	*Could be used to enter data and find relationships.*
	Measurement Based Data	*Could be done at stations or individually.*
Bridge to the Classroom		*This activity transfers directly to the classroom with the only modifications being the settings addressed above.*

Dome Floor Dilemma

Explore

Posing the Problem:

The diagram below represents the tile pattern on the circular floor of a domed building. Each shade, light, medium, and dark, represents a different color of floor tile. Each central angle is congruent to all others.



If you know the length of the radius of the circular floor, is it possible to calculate the area of each shaded region?

Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, you will construct a geometric figure then collect and analyze data. You will determine three functional relationships: area of a sector of a circle versus the radius, area of a segment of a circle versus the radius, and the area of the triangle bound by the segment and the radii drawn to the endpoints of the arc of the segment.

The Sector Construction

For detailed instructions on Geometer's Sketchpad see the **Technology Tutorial T^2--Dome Floor Dilemma.**

1. Construct a circle with a radius.

2. Rotate the radius and the endpoint that lies on the circle 60^o.

3. Construct the intercepted arc of the sector.

4. Construct the interior of the sector.

5. Measure the length of the radius and the area of the sector.

6. Create a table to compare the two measurements. Which one is the independent variable and which one is the dependent variable?

7. Plot the two measurements on a graph and turn on the trace feature.

Collect the Data

8. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row, then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

9. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

10. What patterns do you observe in the table?

11. What observations can you make about your graph?

The Arc Segment Construction

1. Construct the arc segment.

2. Change the color of the segment.

3. Measure the area of the segment.

4. Create a table to compare the measure of the area of the arc segment and the length of the radius. Which one is the independent variable and which one is the dependent variable?

5. Plot the two measurements on the graph and turn on the trace feature.

Collect the Data

Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

8. What observations can you make about your graph?

The Triangle Construction

1. Construct the triangle interior.

2. Measure the area of the triangle.

3. Create a table to compare the area of the triangle to the length of the radius. Which one is the independent variable? Which one is the independent variable?

4. Plot the two measurements on the graph and turn on the trace feature.

Collect the Data

5. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

8. What observations can you make about your graph?

Analyze the Data

1. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the sector, *y*.

2. Verify that your function rule models your data. Explain your verification.

3. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the triangle, *y*.

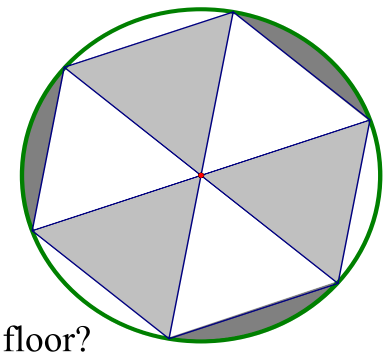
4. Verify that your function rule models your data. Explain your verification.

5. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the segment, *y*.

6. Verify that your function rule models your data. Explain your verification.

7. Recall the floor design discussed earlier. The radius of the circle is 45 feet in length and the cost of tiling the different areas is listed below.

Un-shaded areas - $10.50 per square foot,
Medium shaded areas - $12.00 per square foot and the
Darkest shaded areas - $17.45 per square foot.



Approximately what will be the total cost of tiling the floor?

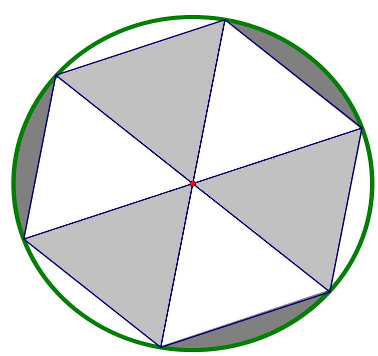
Explain

1. What knowledge of geometric properties was necessary to complete the constructions?

2. What process did you use to develop your algebraic rules?

3. How did you verify your function rules?

4. How did you solve the dome floor dilemma?



5. How can you explain your graph of all three functions in a geometric context?

6. What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?

7. What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?

8. What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?

9. How will the use of these technologies promote a better understanding of the targeted mathematical concepts?

Dome Floor Dilemma
Intentional Use of Data

TEKS		
TEKS		
Question(s) to Pose to Students	Math	
Question(s) to Pose to Students	Tech	
Cognitive Rigor	Knowledge	
Cognitive Rigor	Understanding	
Cognitive Rigor	Application	
Cognitive Rigor	Analysis	
Cognitive Rigor	Evaluation	
Cognitive Rigor	Creation	
Data Source(s)	Real-Time	
Data Source(s)	Archival	
Data Source(s)	Categorical	
Data Source(s)	Numerical	
Setting	Computer Lab	
Setting	Mini-Lab	
Setting	One Computer	
Setting	Graphing Calculator	
Setting	Measurement Based Data	
Bridge to the Classroom		