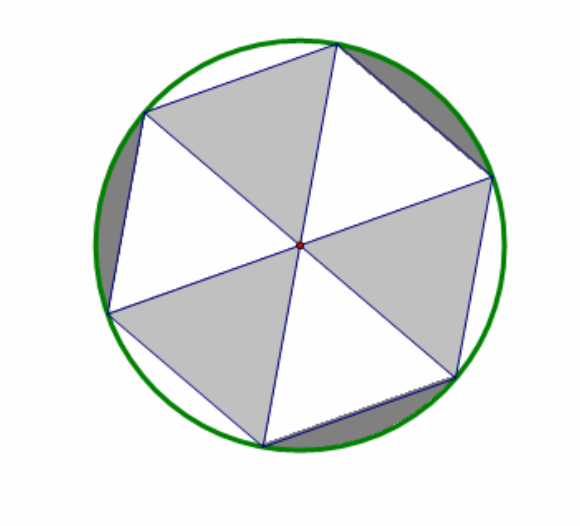
Dome Floor Dilemma

Explore

Posing the Problem:

The diagram below represents the tile pattern on the circular floor of a domed building. Each shade, light, medium, and dark, represents a different color of floor tile. Each central angle is congruent to all others.



If you know the length of the radius of the circular floor, is it possible to calculate the area of each shaded region?

Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, you will construct a geometric figure then collect and analyze data. You will determine three functional relationships: area of a sector of a circle versus the radius, area of a segment of a circle versus the radius, and the area of the triangle bound by the segment and the radii drawn to the endpoints of the arc of the segment.

The Sector Construction

For detailed instructions on Geometer's Sketchpad see the **Technology Tutorial T^2--Dome Floor Dilemma.**

1. Construct a circle with a radius.

2. Rotate the radius and the endpoint that lies on the circle 60^o.

3. Construct the intercepted arc of the sector.

4. Construct the interior of the sector.

5. Measure the length of the radius and the area of the sector.

6. Create a table to compare the two measurements. Which one is the independent variable and which one is the dependent variable?

7. Plot the two measurements on a graph and turn on the trace feature.

Collect the Data

8. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row, then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

9. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

10. What patterns do you observe in the table?

11. What observations can you make about your graph?

The Arc Segment Construction

1. Construct the arc segment.
2. Change the color of the segment.
3. Measure the area of the segment.
4. Create a table to compare the measure of the area of the arc segment and the length of the radius. Which one is the independent variable and which one is the dependent variable?
5. Plot the two measurements on the graph and turn on the trace feature.

Collect the Data

Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.
7. What patterns do you observe in the table?
8. What observations can you make about your graph?

The Triangle Construction

1. Construct the triangle interior.

2. Measure the area of the triangle.

3. Create a table to compare the area of the triangle to the length of the radius. Which one is the independent variable? Which one is the independent variable?

4. Plot the two measurements on the graph and turn on the trace feature.

Collect the Data

5. Click and drag the endpoint of the radius that is on the circle toward the center of the circle until the radius of the circle is approximately 0.5 centimeters. Double click on the table to add another row then click and drag the endpoint of the radius that is on the circle about 0.5 centimeters more away from the center. What do you observe?

6. Double click on the table again, and then move the endpoint of the radius that is on the circle farther away from the center. Repeat this process until you have 8 rows in your table.

7. What patterns do you observe in the table?

8. What observations can you make about your graph?

Analyze the Data

1. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the sector, *y*.

2. Verify that your function rule models your data. Explain your verification.

3. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the triangle, *y*.

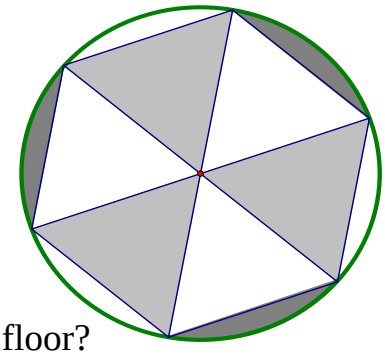
4. Verify that your function rule models your data. Explain your verification.

5. Develop an algebraic rule that describes the relationship of the length of the radius, *x*, to the area of the segment, *y*.

6. Verify that your function rule models your data. Explain your verification.

7. Recall the floor design discussed earlier. The radius of the circle is 45 feet in length and the cost of tiling the different areas is listed below.

Un-shaded areas - $10.50 per square foot,
Medium shaded areas - $12.00 per square foot and the
Darkest shaded areas - $17.45 per square foot.



Approximately what will be the total cost of tiling the floor?

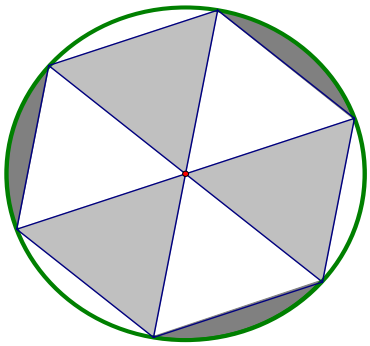
Explain

1. What knowledge of geometric properties was necessary to complete the constructions?

2. What process did you use to develop your algebraic rules?

3. How did you verify your function rules?

4. How did you solve the dome floor dilemma?



5. How can you explain your graph of all three functions in a geometric context?

6. What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?

7. What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?

8. What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?

9. How will the use of these technologies promote a better understanding of the targeted mathematical concepts?

Dome Floor Dilemma
Intentional Use of Data

TEKS		
Question(s) to Pose to Students	Math	
	Tech	
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement Based Data	
Bridge to the Classroom		