Geometric Properties and Sketchpad Skills

Explore Cycle II

Purpose:

Provides participants the opportunity to use dynamic geometry technology to formulate and test conjectures about geometric properties and compare technology use to traditional teaching methods. This part of the training is designed for groups of two, three or four working with a computer station.

Descriptor:

Participants will download pictures from the Internet and/or take digital photos with cameras and import them into Geometer's Sketchpad to explore geometric properties such as parallel and perpendicular lies and planes, congruence, similarity, etc. and make measurements of figures such as perimeter, area, volume. Participants will then use the collected information to formulate and test conjectures about geometric properties. They will then compare this activity with traditional methods of exploring print media with hand-held tolls such as compass, protractors, rulers, etc.

Duration:

2 hours

TEKS:

a(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential I understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

G.7A Use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures.

G.7B Use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons.

G.7C Derive and use formulas involving length, slope, and midpoint.

G.8A Find areas of regular polygons, circles, and composite figures.

G.8B Find areas of sectors and arc lengths of circles using proportional reasoning.

G.8C Derive, extend, and use the Pythagorean Theorem.

G.8D Find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.

G.9A Formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models.

G.9B Formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models.

G.9C Formulate and test conjectures about the properties and attributes of circles and the lines that intersect the based on explorations and concrete models.

G.9D Analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.

TAKS Objectives:

- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 7: Two- and Three-Dimensional Representations of geometric relationships and shapes
- Objective 8: Concepts and Uses of Measurement and Similarity
- Objective 10: Mathematical Processes and Tools

Technology:

- Internet access
- Dynamic geometry software (Geometer's Sketchpad)
- Digital camera (optional)

Materials:

Advance Preparation:

- Participant access to computers with Geometer's Sketchpad (latest version update available from http://www.keypress.com/sketchpad) and/or a projection device to use Geometer's Sketchpad as a whole group demonstration tool.
- Sample sketches: Title.gsp, GeoPicExample1.gsp, GeoPicExample2.gsp, GeoPicExample3.gsp found on the CD.

For each participant:

- Ruler
- Protractor
- Copy of a magazine cover
- **Sketchpad Skills Investigation** activity sheet
- **Explore the World with Geometric Properties** activity sheet
- **Geometric Properties and Sketchpad Skills Intentional Use of Technology** activity sheet printed on green paper

For each group of 2 participants:

- Computers with Geometer's Sketchpad and Microsoft Excel
- Copy of the Technology Tutorial T^2

Geometric Properties and Sketchpad Skills—Leader Notes

1. *Hand out a copy of the cover of a magazine, i.e. a copy of* Mathematics Teacher *from NCTM. Prompt participants to find an example of parallel lines and use measuring tools to prove the lines are parallel.*

2. *Show participants the same magazine cover pasted into Geometer's Sketchpad with parallel lines constructed on top of the letters and proved using the electronic measurement tools. For a sample see the* ***Title*** *sketch.*

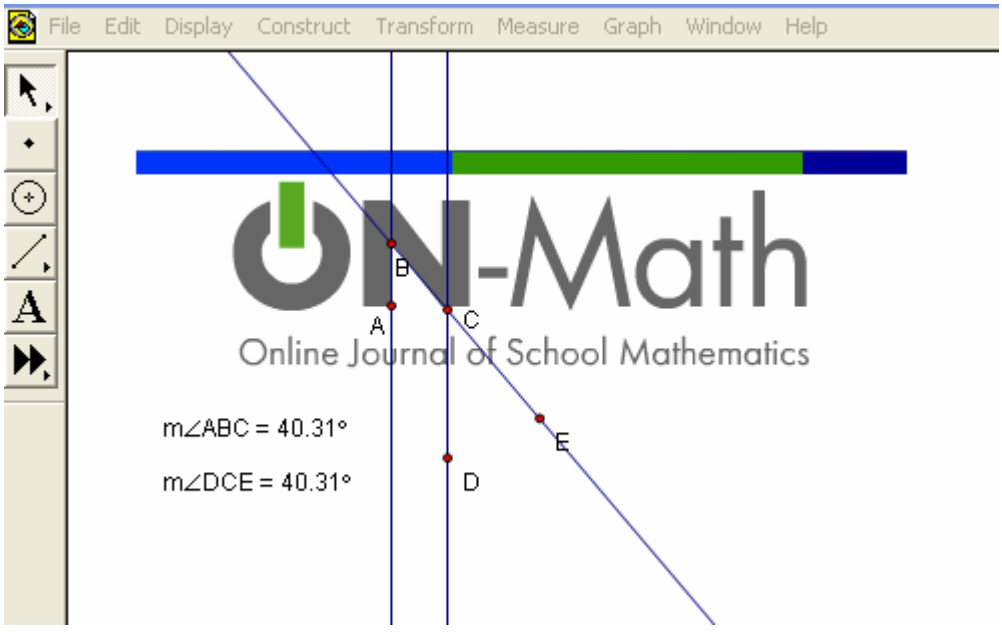
3. *Hand out the* ***Sketchpad Skills Investigation*** *activity sheet. Float among the participants to give assistance as needed. In particular, use facilitation questions to guide participants when they encounter the instructions to measure an angle and measure the area. Participants may use the Technology Tutorial T^2 if they need detailed instructions.*

4. *Hand out the* ***Explore the World with Geometric Properties*** *activity sheet.*

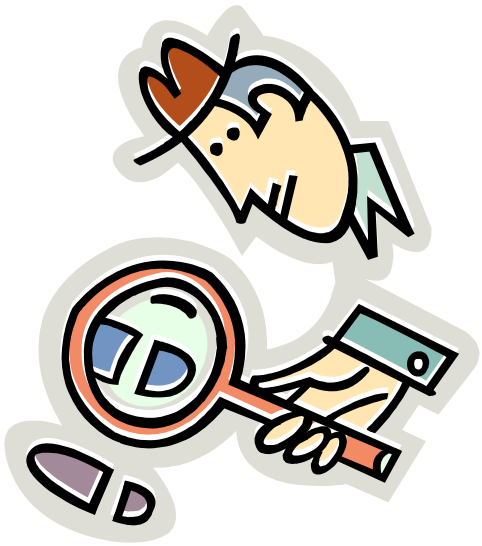
5. *Participants will search the Internet for pictures or take digital photos that represent geometric properties, import them into Geometer's Sketchpad, and then prove the properties using the skills they have just discovered in the Sketchpad Skills Investigation activity.*

Sample Electronic Journal Cover with Constructions in Geometer's Sketchpad

See **Title** sketch.



Sketchpad Skills Investigation



For detailed instructions see Technology Tutorial T²

1. **Open a blank sketch in Geometer's Sketchpad.**

2. **Create some random points. What do you notice?**

3. **Select some points. Deselect them. How did you do this?**
   Participants must use the Selection Tool and click in the blank white space to deselect items. Participants might confuse the Point Tool with the Selection Tool because it is shaped like an arrow or a "pointer." The Selection Tool is used often when working with Geometer's Sketchpad. Remind participants through out to be sure they have de-selected and/or selected items appropriately.

4. **Label some points. What happens when you use the Label Tool? Why do you think this happens?**
   When the cursor is lined up on the point, the hand turns black. This is so that I know I'm on the right point or item.

5. **Make some circles. How can you deselect the last circle?**
   De-selected the circle by clicking on the Selection Tool then clicking anywhere in the blank space.

6. **Construct some segments, lines and rays. How do they differ from each other? Why?**
   The segment has two endpoints; the ray shows one endpoint but the other end goes off the screen; the line goes off the screen in both directions and shows two points on the line. The representation of lines, rays and segments are true to their geometric definitions.

7. **Label some of the segments, lines and rays you have created.**
   Participants might inadvertently label a line when they really wanted to label a point.

8. **Use the box feature of the** Selection **tool to quickly select some of your items. What happened?**
   It selected every item it came in contact with even if it didn't surround it.

9. **Use the Selection Tool to clear all objects from this page.**
   This allows the participants to clear a fresh start to explore the Menu Bar.

10. **Click on the File menu and read the options. Slide the cursor across the menu bar and read the other options. What do you notice about some of the option choices?**
    Not all of the options are available. Things are "grayed" out. This is because nothing is selected.

11. **Draw a segment and use the** Measurement **menu to measure it. Did you encounter any problems? If so, what were they?**
    Problems participants might encounter:
    - Nothing highlighted under the measure bar so I had to be sure it was selected.
    - Highlighting the segment and the endpoints will not allow measurements.

    a. **Did you measure a distance or a length? How could you have measured the other?**
       If participants get distance, then they only highlighted the two endpoints. If the participants get length, then they only highlighted the segment.

    b. **What is the difference between distance and length and how Geometer's Sketchpad interprets this?**
       While the value is the same, the distance measures the distance between the two endpoints and the length is the simply the length of the segment.

    c. **Create a line and measure it. Create a ray and measure it. What did you discover?**
       It is impossible to measure a ray and a line because they go off the page toward infinity.

12. **Draw an angle and use the** Measurement **menu to measure it. Did you encounter any problems? If so, what were they?**
    Problems might be highlighting only the sides of the angle only allows length to be measured not the angle. If the entire angle is highlighted the measurement options are turned off.

Facilitation Question

- How do you traditionally name an angle?
  Name the point of the vertex, or name a point on the side, the vertex and a point on the other side.

  This will be the hint needed to measure their angle.

a. **What was required in order for you to be able to measure your angle?**
I had to select three points of the angle with the vertex in the middle.

13. **Click on one side of your angle and adjust the size of your angle. What happens?**
The angle measure changes with the angle.

14. **Construct a circle and use the Measurement menu to explore the various measurement options. What measurements can be made?**
Circumference, area, and radius.

15. **Adjust the size of your circle by clicking on the control point on the circumference and dragging. What happens?**
The measurements change as the size of the circle changes.

16. **Draw a triangle and use the Measurement menu to explore the various measurement options. What measurements can be made?**
We can measure each side and each angle.

a. **Can you measure the perimeter? Is there another way?**
We can add all the measurements of the sides together. The other way is to construct the interior of a triangle, which we will do next, but some participants might already know this.

b. **How can you measure the area? Is there another way?**
We can measure the height if we create a segment that represents the height and then compute the area using the formula. The other way is to construct the interior of a triangle, which we will do next, but some participants might already know this.

17. **Construct the interior of your triangle. Measure the options that are now available. What were they?**
Perimeter and Area.

18. **Change the size of your triangle. What happens to the measurements?**
All the measurements change as the triangle changes.

19. **Draw a right triangle. Try to move it. Does it stay a right triangle? Why or why not?**
Usually a right triangle that has been drawn will not stay a right triangle. To get a right triangle to stay a right triangle when moved, the right angle must be constructed.

20. **Construct a 30-60-90 triangle.**

21. Explore moving your triangle by clicking on various segments and angles. Which objects allow the triangle to stay the same size? Why?
Each segment will allow the triangle to slide on the page but the size stays the same. Also, the vertex that was constructed as an intersection of two lines will move the triangle but the size doesn't change. This has to do with how they were constructed.

a. **Which parts of the triangle allow it to adjust size? Why?**
The vertex at the right angle and the other vertex along the base of the triangle allow the triangle to change sizes.

b. **Will this triangle always stay a 30-60-90 degree triangle no matter how big or small it gets? How do you know?**
Yes, because we constructed a perpendicular and we rotated an angle to form the 30- or 60-degree angle.

22. Create a Hide/Show button to hide your extra construction pieces.

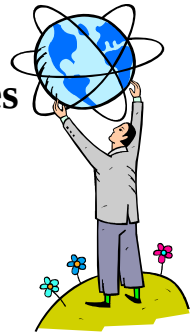
23. Reflect your triangle. What happens?
The entire triangle flipped over the line of reflection

24. How can you continue with this to make a tessellation? Try it.
Continue making transformations.

a. **Did you encounter any challenges? If so, what were they and how did you overcome them?**
Highlighting just one triangle was difficult; I selected the two legs and the vertex that were going to be reflected.

25. What other shapes appear in your tessellation?
Parallelograms, Quadrilaterals, Hexagons…etc.

Explore the World with Geometric Properties



- **Open a new sketch in Geometer's Sketchpad.**

  For detailed instructions on opening a sketch, see the ***Technology Tutorials T²*****.**

- **Search the Internet for pictures or take digital photos that would demonstrate the following geometric properties: parallel lines, tangent to a circle, similar figures, congruent figures, and the central angle of a circle.**
  Challenge: find other geometric concepts represented in the world.

  A search engine such as Google/Image works well with topics such as architecture pictures, kite pictures, bridge pictures, bicycle pictures, etc.

- **Import your pictures into Geometer's Sketchpad, one picture per page.**

  If participants need assistance on importing pictures, there are detailed instructions in the ***Technology Tutorials T²*****.**

- **Use the Geometer's Sketchpad tools to construct and prove the geometric properties represented in your picture. Use a Text Box to show the URL where your picture was found along with any additional information that would be helpful for other participants viewing your construction.**

  Participants may find it difficult to see the default colors or lines as they are constructed on their photos. It may be necessary to remind them change the colors and thickness of figures. For detailed instructions see the ***Technology Tutorials T²*****.**

- **Report your findings to the rest of the participants via the method suggested by the facilitator.**

  See the Explain section below.

Explain

This Explain phase of the professional development provides each group of participants the opportunity to report their discoveries to their peers. This part of the training is designed for each group to present to the entire group while the facilitator encourages participants to see as many mathematical connections as possible. Use facilitation questions to lead the discussion.

Various methods of reporting out:

- *Have each group save their sketch to a thumb drive. Re-open it at a computer connected to the presentation equipment in the room.*
- *Have participants do a gallery tour where one representative of the group stands by their computer to answer questions as other participants come by to look.*
- *Have entire groups rotate until they have viewed all the other sketches and return to their own station.*

*Use the facilitation questions as participants are reporting out or if they are floating from station to station. Ask the questions at the end with a sketch of your own or the example sketch **GeoPicExample**.*

Facilitation Questions

- Did the perspective of the picture cause any problems?
- Which pictures were better then others? Why?
- What were the mathematical ideas that were explored? Were there any others?
- How did you verify the concepts were true?
- What other ways could students have explored the same concepts?
- Were there relationships that you hadn't thought of before? If so, what were they?
- Do you agree with the findings of the other groups?
- What else could you have done with the pictures?
- How will your students react to an activity like this?
- Could you make this more challenging for you students? How?
- What prior knowledge did you need to explore your picture geometrically?
- What questions could you ask students to help them focus on a specific geometric topic as they explored different pictures?
- How would this activity been different if you could import a picture you have taken yourself using a digital camera?
- Was there an underlying theme of mathematical topics in the pictures explored? If so, what was it? Were there sub categories into which the topic could be divided? If so, what are they?

Geometric Properties and Sketchpad Skills

Intentional Use of Data—Leader Notes

1. *At the close of the Explain phase, distribute the* ***Intentional Use of Data*** *activity sheet to each participant.*

2. *Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.*

Facilitation Questions

- Which TEKS formed the primary focus of this activity?
- What additional TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. *As a whole group, share responses for two to three minutes.*

4. *As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.*

Facilitation Question

- What attributes of the activity support the level of rigor that you identified?

5. *As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.*

Facilitation Questions

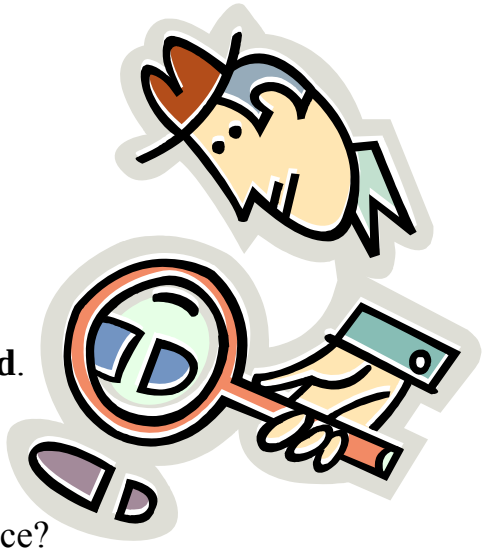
- How would this activity change if we had access to one computer per participant?
- How would this activity change if we had access to one computer per small group of participants?
- How would this activity change if we had access to one computer for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- Why was technology withheld during the first part of this activity (the magazine cover)?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

6. *Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets provide prompts for generating attributes of judicious users of technology during the elaborate pahse.*

Geometric Properties and Sketchpad Skills
Intentional Use of Data *(possible participant answers)*

TEKS		*a(5), a(6),G.7A, G.7B, G.7C, G.8A, G.8B, G.8C, G.9A, G.9B, G.9C, G.9D*	
Question(s) to Pose to Students	Math	*How many different geometric properties were you able to identify in one picture?* *What type of patterns did you discover? Are there others?*	
	Tech	*How did technology help you with the identification of geometric properties?*	
Cognitive Rigor		Knowledge	√
		Understanding	√
		Application	√
		Analysis	√
		Evaluation	√
		Creation	√
Data Source(s)		Real-Time	*none*
		Archival	*none*
		Categorical	*none*
		Numerical	*none*
Setting		Computer Lab	*Each student uses the computer.*
		Mini-Lab	*In groups students take turns or groups switch out.*
		One Computer	*A student operates the control as other students read directions, entire class records data.*
		Graphing Calculator	*Could be used to enter data and find relationships.*
		Measurement Based Data	*Could be done at stations or individually.*
Bridge to the Classroom		*This activity transfers directly to the classroom with the only modifications being the settings addressed above.*	

Sketchpad Skills Investigation



For detailed instructions see Technology Tutorial T²

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11. Draw a segment and use the **Measurement** menu to measure it. Did you encounter any problems? If so, what were they?

    a. Did you measure a distance or a length? How could you have measured the other?

    b. What is the difference between distance and length and how Geometer's Sketchpad interprets this?

    c. Create a line and measure it. Create a ray and measure it. What did you discover?

12. Draw an angle and use the **Measurement** menu to measure it. Did you encounter any problems? If so, what were they?

    a. What was required in order for you to be able to measure your angle?

13. Click on one side of your angle and adjust the size of your angle. What happens?

14. Construct a circle and use the **Measurement** menu to explore the various measurement options. What measurements can be made?

15. Adjust the size of your circle by clicking on the control point on the circumference and dragging. What happens?

16. Draw a triangle and use the **Measurement** menu to explore the various measurement options. What measurements can be made?

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    b. How can you measure the area? Is there another way?

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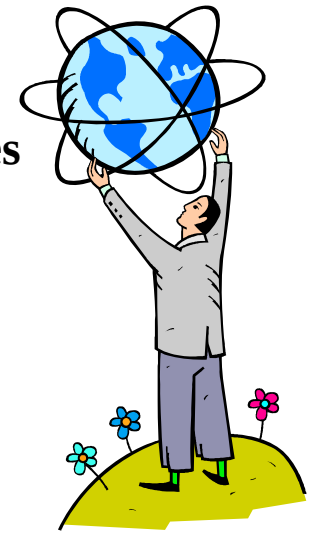
Explore the World with Geometric Properties



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Geometric Properties and Sketchpad Skills
Intentional Use of Data

TEKS		
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	Tech	
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement Based Data	
Bridge to the Classroom		