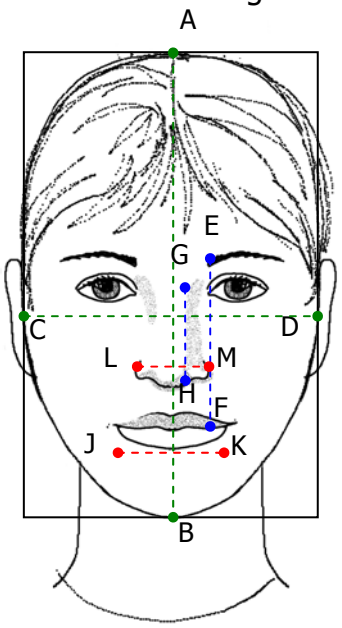
The Eye of the Beholder

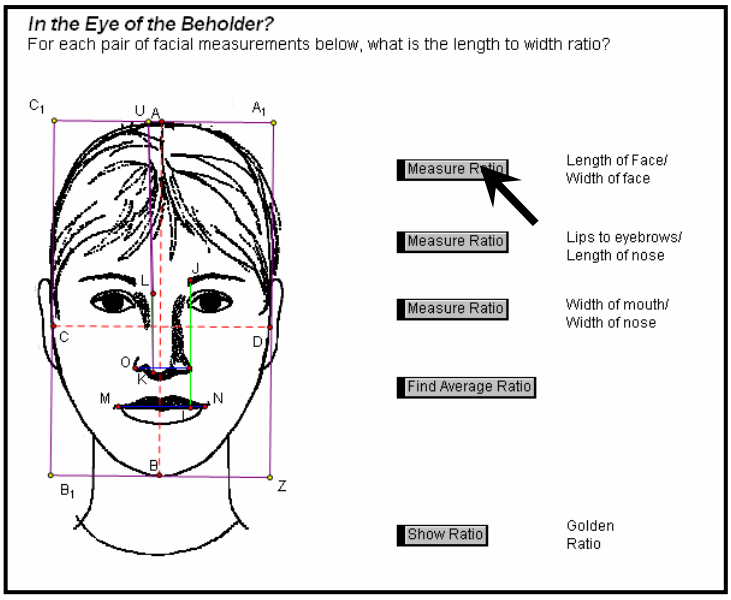
1. Study the features on the artist's sketch below. Identify the segments that represent each of the following ratios.



Sketch by artist, Debra L. Hayden, 2005.
Used by permission

Ratio	Segments
Length of face to Width of face	$\frac{\text{Length of face}}{\text{Width of face}} =$
Lips to eyebrows to Length of nose	$\frac{\text{Lips to eyebrows}}{\text{Length of nose}} =$
Width of mouth to Width of nose	$\frac{\text{Width of mouth}}{\text{Width of nose}} =$
Average Ratio	

2. Open the sketch **Face Sample** in Geometer's Sketchpad. Calculate the ratio values indicated in the sketch by clicking on the "Measure Ratio" action button for the given ratio. Also, calculate the average ratio. Record your answers in your table.



3. Log on to the Internet and open the website http://www.angelfire.com/celeb2/celebrityfaces/. Search for a photo of your favorite celebrity. The photo must be a full front view of the face.

4. Right click on the face and select "Copy" so that you can "insert" the photo into Geometer's Sketchpad.

5. Using Geometer's Sketchpad, construct and measure segments of the face you copied as shown on the sample. Measure the appropriate ratios and record them in the chart below.

I used a photo of :			
Length of face		Ratio	
Width of face			
Lips to eyebrows		Ratio	
Length of nose			
Width of mouth		Ratio	
Width of nose			

6. How do your ratios compare with those found by other groups in the class? Why do you think this is so?

Creating a "Golden" Exponential Function

1. Open the sketch **golden triangle1** to find possible measurements for each of the following:

   The length of $\overline{BC}$ = _____

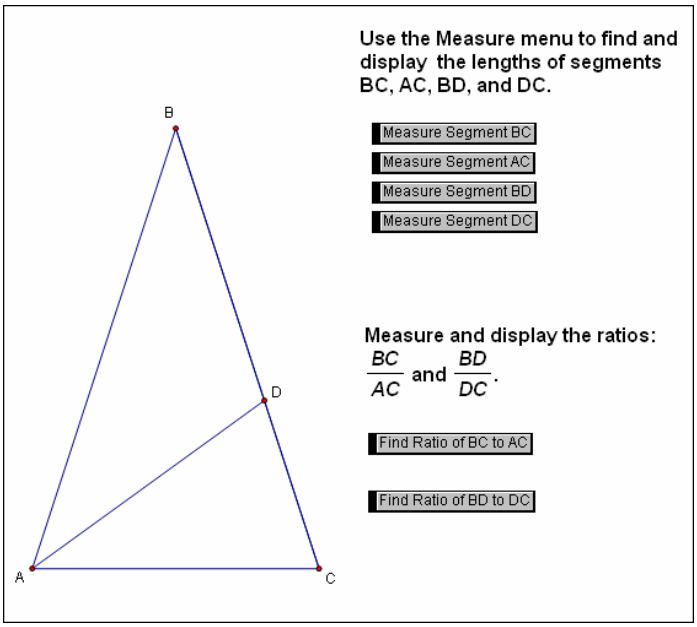
   The length of $\overline{AC}$ = _____

   The ratio of $\overline{BC}$: $\overline{AC}$ ≈ _____

   The length of $\overline{BD}$ = _____

   The length of $\overline{DC}$ = _____

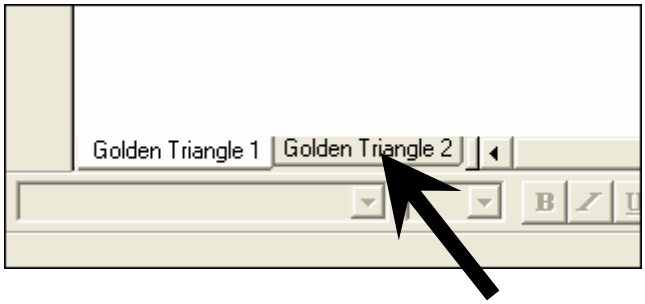
   The ratio of $\overline{BD}$: $\overline{DC}$ ≈ _____

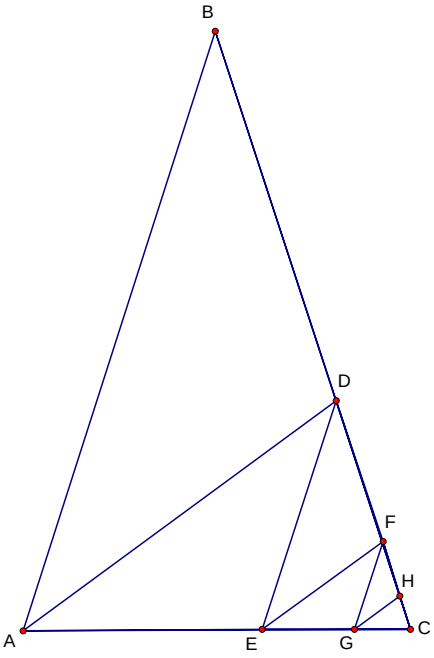


2. Click on Point *C* and drag it around the screen. What happens to the segment lengths?

3. What happens to the ratios when you drag point *C* around the screen?

4. Click the **Golden Triangle 2** tab inside your sketch. Find possible values for the following:





Triangle	Leg	Length	Successive Ratios
1	$\overline{HC}$		
2	$\overline{GC}$		
3	$\overline{FC}$		
4	$\overline{EC}$		
5	$\overline{DC}$		
6	$\overline{AC}$		
7	$\overline{BC}$		

5. Enter the numbers 1 – 7 into List 1 on your graphing calculator. Enter the lengths of the segments $\overline{HC}$, $\overline{GC}$, $\overline{FC}$, $\overline{EC}$, $\overline{DC}$, $\overline{AC}$, and $\overline{BC}$ into List 2. Create a scatter plot on your graphing calculator with the Triangle number on the x-axis and the Leg length on the y-axis. Sketch your plot and describe your window.

6. Determine an exponential function that passes through these points. Explain how you determined the function.

7. Sketch your plot and function graph. Does the function fit the data well? How do you know?

8. What does the coefficient in your function represent in the golden triangle? How did you obtain this value?

9. What does the base of the power in your function represent in the golden triangle?

Algebra and the Golden Ratio

You have found the exact value of the golden ratio to be $\frac{1+\sqrt{5}}{2}$. Let's look at how this value connects to the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, …

Consider the table below but don't fill in the right-hand column until you've answered questions 1 – 3.

Term number	Fibonacci number
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

1. If you made a scatter plot of Fibonacci number vs. term number, what would the scatter plot look like?

2. If you started with 1 as your first Fibonacci number, could you write a function that would pass through all of the points in your scatter plot?

3. How could you make a scatter plot that more closely fits an exponential function?

4. Fill in the table with the Fibonacci numbers of your choice and write an exponential function to fit your points.

5. Which would give a better fit: starting with 5 or starting with 13? How does choosing a different starting number affect your function rule?

The Golden Ratio in Art and Architecture

Search the Internet using key words "golden ratio" and "art" or "architecture." Find one example of how the golden ratio is used in art and one example of its use in architecture. Record at least the following information for each example.

Art Example

The artist is/was _____

The name of the painting, sculpture, etc. is _____

Give a brief description or simple sketch of how the golden ratio is used in this work.

Architecture Example

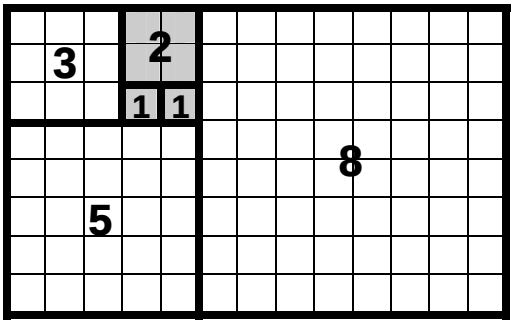
The architect is/was _____(or give country where it is located)

The name of the painting, sculpture, etc. is _____

Give a brief description or simple sketch of how the golden ratio is used in this structure.

Golden Areas

Consider the squares that make up a golden rectangle shown below. The squares have sides that are terms of the Fibonacci sequence: 1, 1, 2, 3, 5,… Each golden rectangle, such as the square that is shaded, is formed by attaching the next Fibonacci square to the previous golden rectangle.



1. Complete the table below to show the relationship between the number of a square in each golden rectangle and the area of the square. Let the 3×3 square be square #1.

Square Number	Area of Square
1	3^2
2	5^2
3	
4	
5	
6	
7	

2. Enter the square numbers into List 1 of a graphing calculator and the areas into List 2. Make a scatter plot for squares 1 – 7. Sketch your scatter plot below and describe the domain and range of the plot.

3. Without using the regression feature on your calculator, write a function that fits your data. Enter your function into your calculator to test it. Alter the function as needed until you are satisfied that it fits the data.

4. Explain how the numbers in your function are related to the data.

5. Would your function be any different if you started with 2^2 instead of 3^2 as the first area? If so, how and why?

1 A stationery company makes cards and posters using dimensions of golden rectangles. So far their inventory includes posters with dimensions (in inches) of 3×5, 5×8, and 8×13. Which equation below would be useful in approximating the length of a poster with a width of 21 inches?

A $L = 13 \times 21$

B $L = 3 \times 1.6^4$

C $L = 13 + 21$

D $L = (1.6)(21)$

2 The table below shows a section of the Fibonacci sequence.

Term number *x*	**Fibonacci number *y***
0	5
1	8
2	13
3	21
:	:

Which function best fits the data shown in the table?

A $y = 1.6x$

B $y = 5 * 1.6^x$

C $y = x^{1.6}$

D $y = 8 * 1.6^x$

3 The exact value of *phi*, referred to as the golden ratio, can be found by taking the larger root of the equation $x^2 = x + 1$. What is the exact value of *phi*?

A $\frac{5}{3}$

B 1.618

C $\frac{1+\sqrt{5}}{2}$

D $\frac{1-\sqrt{5}}{2}$

4 The function $y = 2(1.62)^x$ produces the table below when the domain is {1, 2, 3, …}.

X	Y1
1	3.24
2	5.2488
3	8.5031
4	13.775
5	22.315
6	36.151
7	58.565

X=1

Which function will produce the table

X	Y1
1	8.5031
2	13.775
3	22.315
4	36.151
5	58.565
6	94.875
7	153.7

X=1

for the same domain?

A $y = 1.2346 * 1.62^x$

B $y = 3.24 * 1.62^x$

C $y = 5.2488 * 1.62^x$

D $y = 8.5031 * 1.62^x$