2A.1A	The student is expected to identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.
2A.1B	The student is expected to collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
2A.3A	The student is expected to analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems.
2A.3B	The student is expected to use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.
2A.3C	The student is expected to interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

Materials

Advanced Preparation:

- Student/teacher access to computers with TI Interactive and/or a graphing calculator and a projection device to use TI Interactive as a class demonstration tool.
- Chart paper
- Markers
- Student and group copies of handouts

For each student:

- Graphing calculator
- Student worksheets, *Casey's Part – Time Jobs, Selected Response Questions*
- Access to TI Interactive and TI Interactive Video Rental Sketches 1, 2, 3, and Video Rental Spreadsheet 1

For each student group of 3 to 4 students:

- Chart paper
- Markers
- Group worksheets *Andy and Beca, Video Joe, Shipping Costs*
- Access to TI Interactive and TI Interactive Video Rental Sketches 1, 2, 3, and Video Rental Spreadsheet 1

ENGAGE

The Engage portion of the lesson is designed to create student interest in the topic of video rentals, which will be explored later in the lesson as linear programming. This part of the lesson is designed for groups of three to four students.

1. Have students in groups of 3 or 4.
2. Distribute one copy of *Andy and Beca* to each group of students.
3. Working together, each group should complete the worksheet.
4. Each group will prepare a poster on their responses to Questions 12 – 14.
5. Post the chart papers around the room. Each group should choose one person from the group to stay with the group poster and explain their responses. Have the groups conduct a gallery walk to see all the other posters and solution strategies.
6. Come back together as a large group and debrief. Possible facilitation questions are shown below.

Facilitation Questions – Engage Phase

- How were the posters/solutions the same?
  Answers may vary.
- How were the posters/solutions different?
  Answers may vary.
- What were some of the strategies used?
  Answers may vary. Some groups may have solved the problem graphically, some may have used a table containing a list of the possible outcomes, some may have used logical reasoning to solve the problems.
- Did you see any posters/solutions that included graphs? How were they used?
  Answers may vary. If a student group used a graph to solve the problem, they likely have a shaded region bounded by the lines $x = 5$, $y = 7$, and $x + y = 10$.
- Did you see any posters/solutions that included tables? How were they used?
  Answers may vary. If a student group used a table, they may have a 3-column table: number of VHS tapes, number of DVDs, and total cost.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for students to work in groups of 3 to 4 students.

1. Distribute copies of *Video Joe* to each group of students.
2. Have groups work through *Video Joe*.
3. When finished, each group should record their solutions on chart paper, answering all the questions included in the situation.
4. Have each group post their chart paper.

Facilitation Questions – Explore Phase

- How does the amount of shelf space limit the number of DVDs and VHS tapes Joe can stock?
  He can only stock 1500 inches worth of videos.
- What other information in the problem limits the number of DVDs and VHS tapes he can stock?
  Twice as many VHS as DVD, between 80 and 200 VHS
- What happened to the region as more inequalities, or restrictions, were applied?
  The region became more limited.
- What does this limited region mean in the situation?
  Each restriction in the situation limits the number of VHS tapes and DVDs that can be stocked.
- Do all the charts have the same feasible regions shown? How are they the same or different?
  Graphs could have different windows/domain and range settings or different scaling.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief the *Video Joe* activity. Use group posters when asking the Facilitation Questions.

Facilitation Questions – Explain Phase

- What is the inequality representing the number of VHS tapes and DVDs Video Joe can stock on his shelves? $5x + 4y \leq 1500$ Why is the inequality less than 1500 and not greater than 1500? *Video Joe can stock fewer videos, but not more videos that take up 1500 inches of shelf space.* What do the '5' and '4' represent in the situation? *each VHS tape takes up 4 inches of shelf space and each DVD takes up 5 inches of shelf space*
- Could you have graphed the inequality in standard form, by hand? *Yes, by using the x-intercepts and y-intercepts.* What do the *x*-intercept and *y*-intercept represent in the situation? *The x-intercept represents the number of DVDs Video Joe could stock if he had no VHS tapes and the y-intercept represents the number of VHS tapes Video Joe could stock if he had no DVDs.*
- What is the inequality in slope-intercept form? $y = -\frac{5}{4}x + 375$ Why is it important to put the inequality in slope-intercept form? *to plot the inequality quickly into TI Interactive or a graphing calculator.* What does the *y*-intercept represent in the situation? *the y-intercept represents the number of VHS tapes Video Joe could stock if he had no DVDs.*

Facilitation Questions – Explain Phase, continued

- What window settings did you use when graphing the inequality? Why? *Answers will vary. They should range from 0 – 400, approximately. It is important for them to understand why negative values are not necessary in this situation.*
- How did the inequality $y \leq 2x$ restrict the possible combinations of VHS tapes and DVDs Video Joe could stock? *Of the region represented by the first inequality, now the only combinations that can be used are the ones where the number of VHS tapes is at least twice the number of DVDs.*
- How did limiting the number of VHS tapes (between 80 and 200) restrict the possible combinations of VHS tapes and DVDs Video Joe could stock? *The number of possible combinations were decreased and limited to y-values between 80 and 200.*
- What happened to the original region as restrictions were added to the situation? *The original region got smaller each time a restriction was added.*

- The area common to all the restrictions of the situation is called the feasible region. Is the point (50 , 250) inside the feasible region? *No* Explain your answer in terms of the situation. *This point exceeds 200 VHS tapes, so therefore does not fit within the restriction that the number of VHS tapes has to be between 80 and 200.* Is the point (150 , 100) inside the feasible region? *Yes* Explain your answer in terms of the situation. *It meets all the restrictions of the situation.*
- Where is the point (100 , 200) in regards to the feasible region? *At the intersection of two of the inequalities.* How is this point different from the other two points you have looked at? *This point is located at an intersection of two inequalities, and therefore lies on the edge of the feasible region.* This point is one of the vertices of the feasible region. What are the coordinates of the other vertices? *(140 , 200), (236 , 80), (40 , 80).* What do all the vertices points have in common? *They all occur at the intersection of two of the restrictions of the feasible regions.* What are some methods that were used on the graphing calculator or in TI Interactive for finding the coordinates of the vertices? *Table, trace, calculate the intersection, graph*
- When calculating the amount of profit Video Joe could make, are there other combinations of VHS tapes and DVDs that would generate a different amount of profit (other than those listed in the table)? *Yes* Do you think there are any of those combinations that would generate more than $660 or less than $240 in profit? *No* Why or why not? Use the spreadsheet to verify your answer. *Students should enter various points in the spreadsheet trying to generate more than $660 or less than $240. By using the spreadsheet portion of TI Interactive, the students can readily see the profit being generated by the different combinations of VHS tapes and DVDs. The vertices of the feasible region contain the extremes of the region, therefore from those points you will find the maximum/minimum profit for Video Joe's store.*

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for students working in small groups.

1. Distribute **Shipping Costs** to each group of students. Divide students into an even number of groups.
2. All groups should have access to TI Interactive or a graphing calculator.
2. Have groups work together and record their actions on chart paper. Students should include all aspects of solving the problem on the chart paper and include sketches of the feasible region, inequalities, and justification of their solution using the cost function.
3. Put two groups together and have them explain to each other how they arrived at their solutions. After both groups have explained their strategies, the two groups should decide what their two posters have in common and how are they different.
4. Each pair of groups will present a summary to the whole class of the various strategies used, what their two posters had in common and how they were different. Use the Facilitation Questions to debrief the activity.

Facilitation Questions – Elaborate Phase

- How is this situation different from *Video Joe*?
  This is asking for a minimum cost vs. maximum profit in Video Joe
- What were some of the things all the posters had in common?
  Answers may vary.
- What were some of the differences between the posters?
  Answers may vary.
- Describe the restrictions or limitations in this situation.
  Truck capacity, capacity of each case, minimums/maximums of types of videos needed
- What were the window settings for the graphs? Why were these values chosen?
  Answers may vary. Window settings should be chosen that are appropriate to the domain and range of the situation.
- Did you have to adjust the window settings of your graph as you added restrictions to the graph? Why?
  Answers may vary. Depending on the original window chosen, sometimes adding restrictions will force students to zoom in on a certain region or to enlarge their window to see new vertex points.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute *Casey's Part-time Jobs* to each student.
2. Each student should complete the assessment, showing all appropriate work.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	2A.1A	B	A	C	D		
2	2A.3A	A	C	D	B		
3	2A.3C	C	A	B			D
4	2A.3B	B	C	D	A		

Andy and Beca

Andy and Beca are renting videos for the weekend. They can only to afford to rent a maximum of six videos. Some of the videos must be on VHS tapes and some must be on DVD.

1. What are the possible combinations of VHS and DVD Andy and Beca can rent? Use the table to list all the possible combinations.

VHS	DVD
1	*1*
1	*2*
1	*3*
1	*4*
1	*5*
2	*1*
2	*2*
2	*3*
2	*4*
3	*1*
3	*2*
3	*3*
4	*1*
4	*2*
5	*1*

Open **Video Rental Sketch 1** through TI Interactive.

2. What does each plotted point represent on the graph?
   Each point represents one possible combination of renting VHS tapes and DVDs.

3. Why are there no points with negative coordinates plotted on the graph?
   You cannot rent a negative amount of tapes of DVDs.

4. Does your table match the table shown on the sketch?
   Answers may vary.

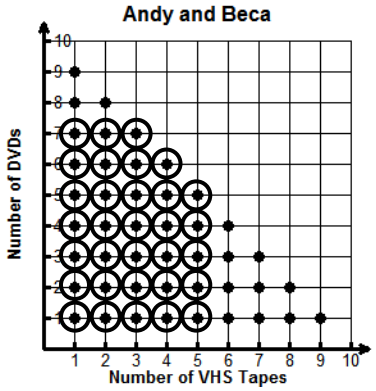
5. Predict how the graph would change if Andy and Beca could rent a total of 10 videos.
   There would be more combinations possible, therefore more points on the graph.

6. Open **Video Rental Sketch 2** to check your prediction. How do the two graphs compare? Explain.
   Answers may vary.

7. What are the total number of combinations of renting VHS tapes and DVDs?
   45 combinations

8. If Andy and Beca limit the number of VHS tapes to 5 or less and the number of DVDs to 7 or less, how would the graph change?
   There would be 13 fewer points, or combinations of VHS tapes and DVDs.

9. Shade the graph below to show the new restrictions to the number of VHS tapes and DVDs Andy and Beca could rent.



10. What are the possible combinations, with the new restrictions included?
    32 possible combinations
11. What are the outermost points of the restricted region?
    (1 , 7) (3 , 7) (5 , 5) (5 , 1) (1 , 1)

12. If VHS tapes rent for $4 and DVDs rent for $2, what is the most they could spend if they stay within all the restrictions? What combination of VHS tapes and DVDs would that be?
    $30 for 5 VHS tapes and 5 DVDs.

13. If VHS tapes rent for $3 and DVDs rent for $4, what is the most they could spend? What combination of VHS tapes and DVDs would that be?
    $37 for 3 VHS tapes and 7 DVDs.

14. How did the cost change from the first situation to the second situation? Why?
    An increase of $7; VHS tapes cost less and DVDs cost more in the second situation.

Video Joe

Video Joe has decided to open a small video rental store. He plans on offering DVDs and VHS tapes for rental. After installing all the shelves in the store, he calculates that he has 125 feet of shelf space to store the DVDs and VHS tapes. Each DVD takes up 5 inches of shelf space, while each VHS tape takes up 4 inches of shelf space.

Let x = the number of DVDs and y = the number of VHS tapes he can stock on his shelves at any given time.

1. Write an equation describing the number of VHS tapes and DVDs Video Joe can stock on his shelves, given the limited amount of shelf space.
   $5x + 4y = 1500$ *125 feet = 1500 inches*

2. Would Video Joe be able to stock more or less VHS tapes and DVDs than represented by the equation? Justify your answer.
   Less. With a defined amount of shelf space, Video Joe can always put fewer videos on the shelves, but it is impossible to put more.

3. Write the equation as an inequality to represent this situation.
   $5x + 4y \le 1500$

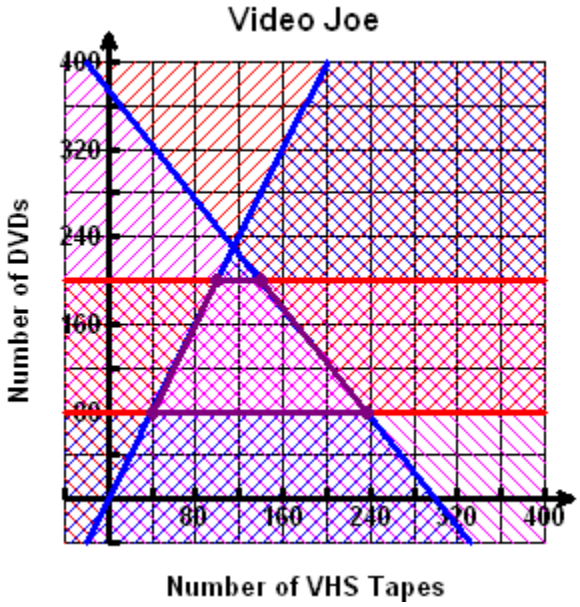
4. Write the inequality in slope – intercept form.
   $5x + 4y \le 1500$
   $4y \le -5x + 1500$
   $y \le -\frac{5}{4}x + 375$

5. Graph this inequality on TI Interactive or graphing calculator. Describe the region that would apply to this inequality.
   The x-intercept is (300 , 0) and the y-intercept is (0, 375). The region under the line applies to this inequality since Video Joe can use less than 1500 inches of shelf space, but not more.

6. Video Joe would like to keep at most 2 times as many VHS tapes as DVDs. Write an inequality to represent this restriction.
   $y \le 2x$
7. Graph this inequality on TI Interactive or graphing calculator on the same screen as the previous inequality. Describe the region that now applies to the two restrictions (inequalities).
   The common area that is located beneath each of the lines.

8. Video Joe would also like to keep between 80 and 200 VHS tapes in stock. Write two inequalities to represent this restriction.
   $y \geq 80$
   $y \leq 200$ or $80 \leq y \leq 200$
9. Graph these two inequalities on TI Interactive or graphing calculator on the same screen as the two previous inequalities. What region represents all the restrictions, or inequalities, in this situation?
   The common area beneath the first two inequalities and between $y = 80$ *and* $y = 200$*.*

10. Open **Video Rental Sketch 3**. How does your graph compare to this one? Explain. (The purple trapezoidal region represents the region common to all restrictions.)
    Answers may vary.



11. What are the vertices of the region common to all the restrictions (**feasible region**)?
    (100 , 200) (140 , 200) (236 , 80) (40 , 80)

12. What do these coordinates represent in this situation?
    100 DVDs, 200 VHS tapes
    140 DVDs, 200 VHS tapes
    236 DVDs, 80 VHS tapes
    40 DVDs, 80 VHS tapes

13. Video Joe makes a profit of $2.25 on each DVD rented and $1.50 on each VHS tape rented. Write a function representing the profit he makes if he rents x number of DVDs and y number of VHS tapes.

$f(x,y) = 2.25x + 1.50y$

14. Use the profit function to determine the amount of profit Video Joe would make using the coordinates of the feasible region.
Students could use a calculator or the spreadsheet feature on TI Interactive to calculate each value.

15. Use the spreadsheet in TI Interactive to enter the coordinates of the feasible region and the profit function. Open **Video Rental Spreadsheet 1** to verify your answers. How do these answers compare with yours? Explain any differences.
Answers may vary.

DVDs	VHS	Profit
100	200	600
140	200	660
236	80	534
40	80	240

16. Which combination would generate the most profit for Video Joe, but still meet all the restrictions? How do you know?
140 DVDs and 200 VHS tapes will generate $660 for Video Joe. Using a trial-and-error approach, students will not find another combination of DVDs and VHS tapes that will generate profit greater than $660.

Shipping Costs

Video Joe orders all his DVDs and VHS tapes from an area supplier. The supplier has only one truck available for delivery and it has a capacity of 3600 cubic feet. One case of VHS tapes takes up 18 cubic feet of space, while one case of DVDs takes up 12 cubic feet of space. Video Joe places an order with the supplier for one truckload of VHS tapes and DVDs. He has to order between 150 and 240 cases of DVDs to meet the demand and at least 20 cases of VHS tapes. The shipping costs are based on the number of cases on the truck. Each case of VHS tapes costs $3.50 in shipping costs and each case of DVDs costs $3.75 in shipping costs.

Let x = number of cases of VHS tapes
y = number of cases of DVDs

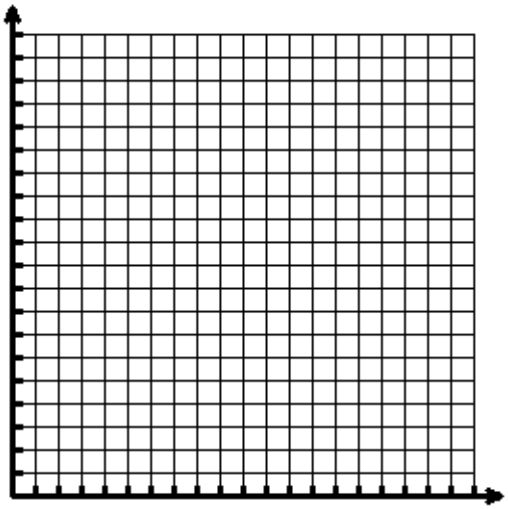
How many cases of VHS tapes and DVDs should he order if he would like to pay the least amount possible in shipping costs and stay within all the restrictions?
20 cases of VHS tapes and 150 cases of DVDs

Casey's Part-time Jobs

Casey the college student is working two part-time jobs. He works at a video rental store for $5.25 per hour and at a movie theatre for $6.05 per hour. He wants to work no more than 30 hours per week. He wants to work between two and three times the hours at the movie theatre than at the video store. He also has to work a minimum of 10 hours per week at the movie theatre.

Let x = the number of hours worked at the video store
y = the number of hours worked at the movie theatre

Use TI-Interactive or a graphing calculator to graph the feasible region described above. Record the feasible region below. Label the axes and the vertices of the feasible region.



How many hours should he work at each job to earn the maximum amount of money each week?
What is the maximum amount of money he could make each week? Justify your answers.
He should work 10 hours a week at the video store and 20 hours a week at the movie theatre.
$173.50

Andy and Beca

Andy and Beca are renting videos for the weekend. They can only to afford to rent a maximum of six videos. Some of the videos must be on VHS tapes and some must be on DVD.

1. What are the possible combinations of VHS and DVD Andy and Beca can rent? Use the table to list all the possible combinations.

VHS	DVD

Open **Video Rental Sketch 1** through TI Interactive.

2. What does each plotted point represent on the graph?

3. Why are there no points with negative coordinates plotted on the graph?

4. Does your table match the table shown on the sketch?

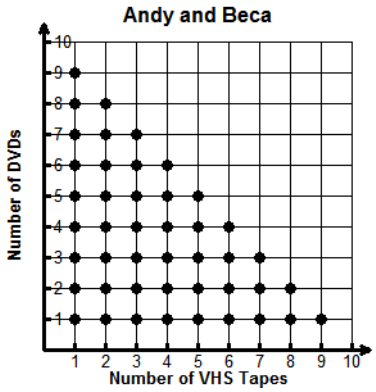
5. Predict how the graph would change if Andy and Beca could rent a total of 10 videos.

6. Open **Video Rental Sketch 2** to check your prediction. How do the two graphs compare? Explain.

7. What are the total number of combinations of renting VHS tapes and DVDs?

8. If Andy and Beca limit the number of VHS tapes to 5 or less and the number of DVDs to 7 or less, how would the graph change?

9. Shade the graph below to show the new restrictions to the number of VHS tapes and DVDs Andy and Beca could rent.



10. What are the possible combinations, with the new restrictions included?

11. What are the outermost points of the restricted region?

12. If VHS tapes rent for $4 and DVDs rent for $2, what is the most they could spend if they stay within all the restrictions? What combination of VHS tapes and DVDs would that be?

13. If VHS tapes rent for $3 and DVDs rent for $4, what is the most they could spend? What combination of VHS tapes and DVDs would that be?

14. How did the cost change from the first situation to the second situation? Why?

Video Joe

Video Joe has decided to open a small video rental store. He plans on offering DVDs and VHS tapes for rental. After installing all the shelves in the store, he calculates that he has 125 feet of shelf space to store the DVDs and VHS tapes. Each DVD takes up 5 inches of shelf space, while each VHS tape takes up 4 inches of shelf space.

Let x = the number of DVDs and y = the number of VHS tapes he can stock on his shelves at any given time.

1. Write an equation describing the number of VHS tapes and DVDs Video Joe can stock on his shelves, given the limited amount of shelf space.

2. Would Video Joe be able to stock more or less VHS tapes and DVDs than represented by the equation? Justify your answer.

3. Write the equation as an inequality to represent this situation.

4. Write the inequality in slope – intercept form.

5. Graph this inequality on TI Interactive or graphing calculator. Describe the region that would apply to this inequality.

6. Video Joe would like to keep at most 2 times as many VHS tapes as DVDs. Write an inequality to represent this restriction.

7. Graph this inequality on TI Interactive or graphing calculator on the same screen as the previous inequality. Describe the region that now applies to the two restrictions (inequalities).

8. Video Joe would also like to keep between 80 and 200 VHS tapes in stock. Write two inequalities to represent this restriction.

9. Graph these two inequalities on TI Interactive or graphing calculator on the same screen as the two previous inequalities. What region represents all the restrictions, or inequalities, in this situation?

10. Open **Video Rental Sketch 3**. How does your graph compare to this one? Explain. (The purple trapezoidal region represents the region common to all restrictions.)

11. What are the vertices of the region common to all the restrictions (**feasible region**)?

12. What do these coordinates represent in this situation?

13. Video Joe makes a profit of $2.25 on each DVD rented and $1.50 on each VHS tape rented. Write a function representing the profit he makes if he rents x number of DVDs and y number of VHS tapes.

14. Use the profit function to determine the amount of profit Video Joe would make using the coordinates of the feasible region.

15. Use the spreadsheet in TI Interactive to enter the coordinates of the feasible region and the profit function. Open **Video Rental Spreadsheet 1** to verify your answers. How do these answers compare with yours? Explain any differences.

16. Which combination would generate the most profit for Video Joe, but still meet all the restrictions? How do you know?

Shipping Costs

Video Joe orders all his DVDs and VHS tapes from an area supplier. The supplier has only one truck available for delivery and it has a capacity of 3600 cubic feet. One case of VHS tapes takes up 18 cubic feet of space, while one case of DVDs takes up 12 cubic feet of space. Video Joe places an order with the supplier for one truckload of VHS tapes and DVDs. He has to order between 150 and 240 cases of DVDs to meet the demand and at least 20 cases of VHS tapes. The shipping costs are based on the number of cases on the truck. Each case of VHS tapes costs $3.50 in shipping costs and each case of DVDs costs $3.75 in shipping costs.

Let x = number of cases of VHS tapes
y = number of cases of DVDs

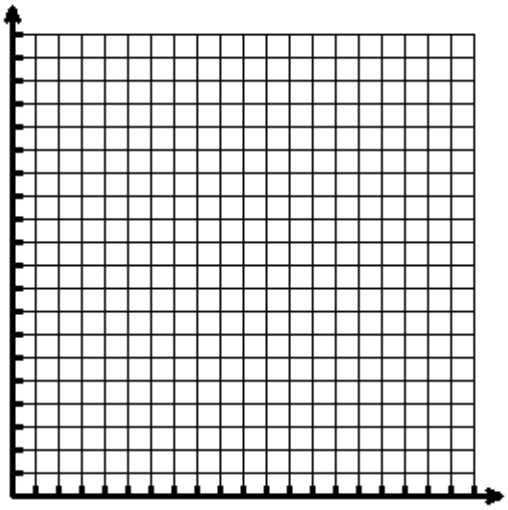
How many cases of VHS tapes and DVDs should he order if he would like to pay the least amount possible in shipping costs and stay within all the restrictions?

Casey's Part-time Jobs

Casey the college student is working two part-time jobs. He works at a video rental store for $5.25 per hour and at a movie theatre for $6.05 per hour. He wants to work no more than 30 hours per week. He wants to work between two and three times the hours at the movie theatre than at the video store. He also has to work a minimum of 10 hours per week at the movie theatre.

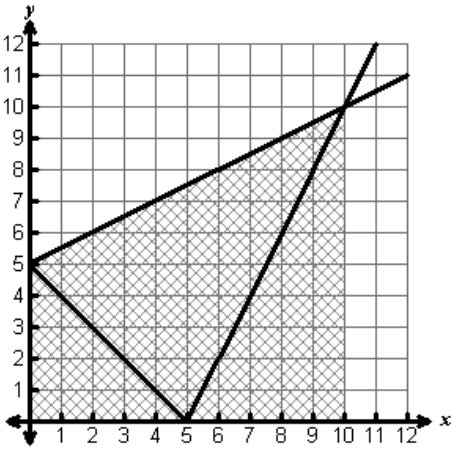
Let x = the number of hours worked at the video store
y = the number of hours worked at the movie theatre

Use TI-Interactive or a graphing calculator to graph the feasible region described above. Record the feasible region below. Label the axes and the vertices of the feasible region.



How many hours should he work at each job to earn the maximum amount of money each week? What is the maximum amount of money he could make each week? Justify your answers.

1 Shown below is a feasible region. The profit function for the region is $f(x,y) = 6x + 5y$.



What are the minimum and maximum values of the function?

A 5 and 10

B 25 and 110

C 25 and 30

D 0 and 110

2 A company machines and sells nuts and bolts. One machine can produce 100 nuts and 50 bolts each hour. Due to demand, the company must produce at least two times more nuts than bolts. The machine runs a maximum of 40 hours per week. Which set of inequalities shows the correct restrictions for this situation if x represents the number of nuts produced in one week and y represents the number of bolts produced in one week?

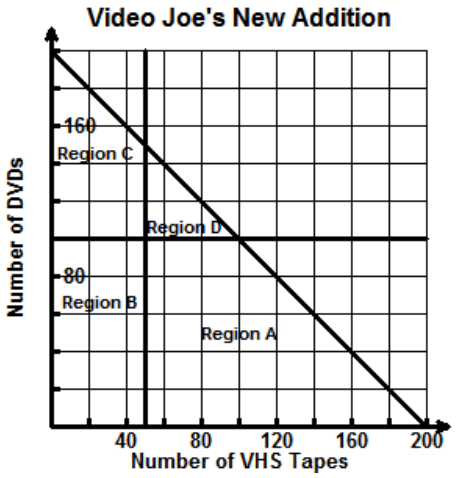
A $\frac{x}{100} + \frac{y}{50} \le 40$, $y \ge \frac{1}{2}x$

B $\frac{x}{50} + \frac{y}{100} \le 40$, $y \ge \frac{1}{2}x$

C $\frac{x}{100} + \frac{y}{50} \le 40$, $y \ge 2x$

D $\frac{x}{50} + \frac{y}{100} \le 40$, $y \ge 2x$

3 Video Joe is expanding his video store. He added enough shelving to hold a maximum of 200 items. He wants to have 50 VHS tapes at most and at least 100 DVDs in stock at all times in the new addition.



Which of the following regions represents the limited restrictions of this situation?

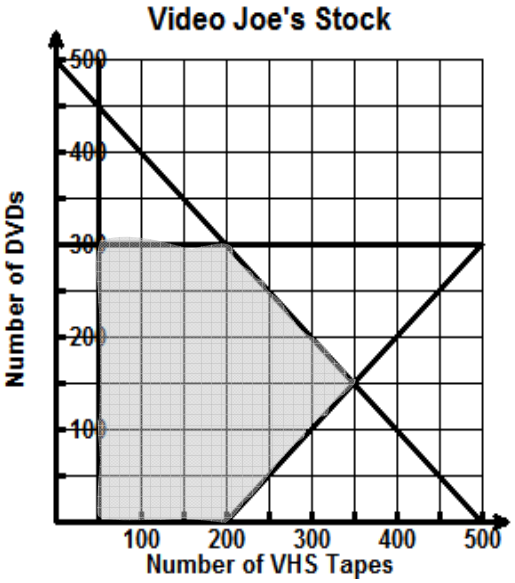
A Region A

B Region B

C Region C

D Region D

4 The feasible region shown below represents the possible amounts of VHS tapes and DVDs on Video Joe's shelves at any given time.



If he makes $1.75 on each VHS rental and $2.00 on each DVD rental, which combination of VHS tapes and DVD rentals would result in the most profit?

A 350 VHS and 150 DVD

B 200 VHS and 300 DVD

C 150 VHS and 350 DVD

D 300 VHS and 200 DVD