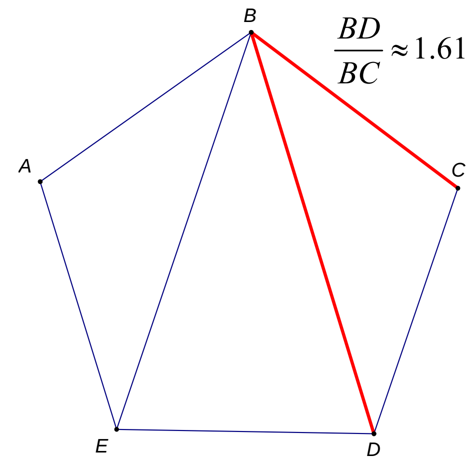
A Golden Idea

Mathematics has been an important influence throughout civilization, from ancient times to the present. In ancient Egypt, Greece, and Rome, geometry and proportion were used in art and architecture. Medieval Europeans carried this tradition of using proportion as they built beautiful cathedrals. Renaissance painters and sculptors used proportion to convey their idea of natural beauty.

Where does this proportion originate? The Greeks found a certain ratio to be prevalent in the natural world around them.

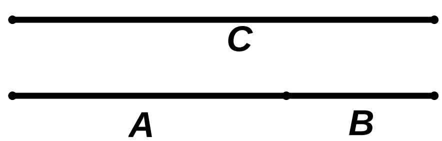
The golden ratio can be developed from several constructions. One way to construct the golden ratio is using the diagonals of a regular pentagon.

In regular pentagon *ABCDE* (shown at right), the ratio of the length of a diagonal from vertex B to the length of a side of the pentagon is always the same. This ratio is called the **golden ratio**, which the Greeks notated with the capital letter *phi*, or Φ.



$$\frac{BD}{BC} \approx 1.618 = \Phi$$

From a segment length perspective, the golden ratio is a geometric mean. Geometrically, segment (in the diagram below, of length *C*) can be split into two smaller segments (of lengths *A* and *B*).

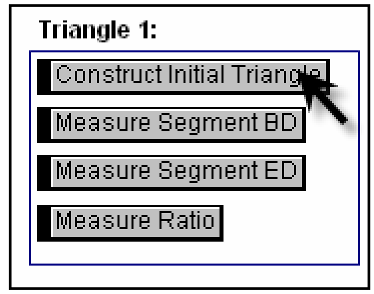


The splitting of the segment is such that the ratio of the length of the original segment to the length of the larger piece (*C*:*A*) is the same as the ratio of the length of the larger piece to the length of the smaller piece (*A*:*B*). In other words,

$$\frac{C}{A} = \frac{A}{B}$$

If the golden ratio is applied in succession to a geometric construction, what types of functional behaviors are present?

Part 1: Investigating Leg Length

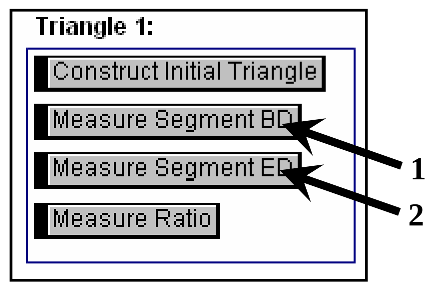


Open the Geometer's Sketchpad sketch "Golden Triangles.GSP." Pentagon *ABCDE* is a regular pentagon. From this regular pentagon, a series of triangles can be constructed. Click on the "Construct Initial Triangle" action button.

1. What kind of triangle is ΔBED? How do you know?

2. Is your triangle classification from Question 1 true for every triangle formed when two diagonals are drawn from one vertex of a regular pentagon? How do you know?

Measure the length of $\overline{BD}$ by clicking on the "Measure Segment BD" action button. Measure the length of $\overline{ED}$ by clicking on the "Measure Segment ED" action button.



3. What is the ratio of the length of $\overline{BD}$ to the length of $\overline{ED}$? How did you find this ratio?

4. What does this ratio represent?

Click on the "Construct Triangle 1" button. This animation bisects angle *BED*, then rotates the resulting triangle 108° to the same orientation as the original triangle. Measure the length of $\overline{CG}$ by clicking on the "Measure Segment CG" button.

5. What is the ratio of $\frac{BD}{CG}$? $\frac{CG}{BD}$? How do these numbers compare?

6. How does ΔCDG compare to ΔBED? How do you know?

7. What scale factor could be applied to ΔBED to generate ΔCDG? Have you seen this ratio before? If so, where?

Click on the "Construct Triangle 2" button. This animation constructs ΔJGK in the same manner as the construction of ΔCDG. Measure the length of $\overline{JK}$ by clicking on the "Measure Segment JK" button.

8. How does ΔJGK compare to ΔCDG? How do you know?

Click the "Construct Triangle 3" button. This animation constructs ΔMKN in the same manner as the construction of ΔJGK. Measure the length of $\overline{MN}$ by clicking the "Measure Segment MN" button.

9. How does ΔMKN compare to ΔJGK? How do you know?

Click the “Construct Triangle 4” button. This animation constructs ΔQNR in the same manner as the construction of ΔMKN. Measure the length of $\overline{QR}$ by clicking the “Measure Segment QR” button.

10. How does ΔQNR compare to ΔMKN? How do you know?

11. What patterns do you observe in the sequence of triangles?

12. Record the measures of the leg of each triangle in the following table.

Triangle Name	Triangle #	Name of Leg	Length of Leg	Process	Ratio
ΔBED					
ΔCDG					$\frac{CG}{BD} =$
ΔJGK					$\frac{JK}{CG} =$
ΔMKN					$\frac{MN}{JK} =$
ΔQNR					$\frac{QR}{MN} =$

13. Record the ratio of each leg length to its previous leg length in the table.

14. Use an appropriate technology to generate a scatterplot of Leg Length vs. Triangle Number (let ΔBED be Triangle Number 0). Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

15. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

16. Use the parent function from Question 15 to determine a function rule to describe the relationship between triangle number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

17. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

18. Compare the domain of your data and the domain of the function rule.

19. Compare the range of your data and the range of the function rule.

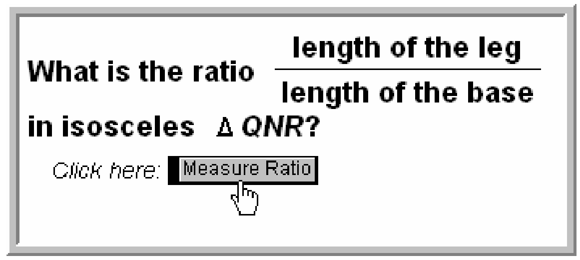
20. What will be the length of the leg of the 9th triangle in this sequence? Explain how you determined your answer.

21. Which triangle will be the first one to have a leg length less than 0.5 cm? Explain how you determined your answer.

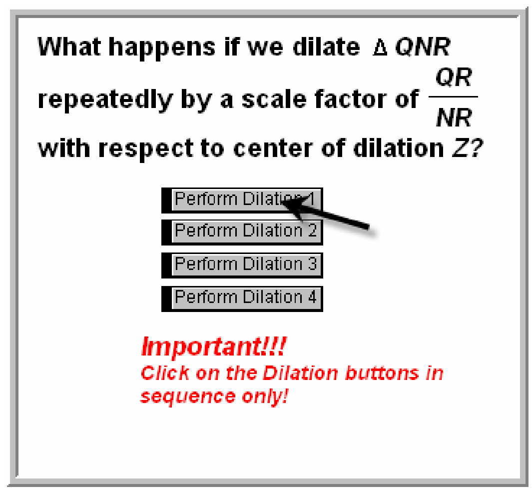
Part 2: Investigating Dilations

In the previous activity, you constructed a series of golden isosceles triangles. What happens if we take a golden triangle and enlarge it repeatedly?

1. In the same Geometer's Sketchpad sketch, click on the "Investigating Dilations" tab. In isosceles ΔQNR, what is the ratio of the length of the leg, QR, to the length of the base, NR?



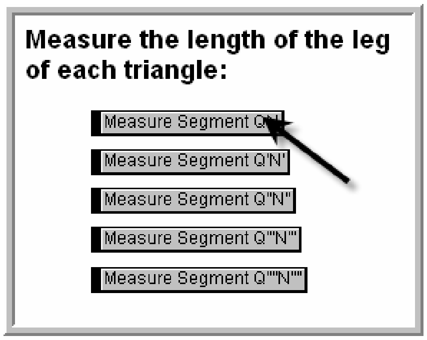
2. Click the "Perform Dilation 1 button." Describe what you see.



3. Click the remaining "Perform Dilation" buttons in sequential order, one at a time. Describe the result.

4. How do each of the triangles compare with each other? How do you know?

5. Measure the leg lengths of each triangle by clicking the "Measure Segment" buttons in order, one at a time. Record the segment lengths in the table below.



Triangle		Name of Leg	Length of Leg	Process	Ratio
Name	Dilation Number				
ΔQNR	*0*				
$\Delta Q'N'N$	*1*				$\frac{Q'N'}{QN} =$
$\Delta Q''N''N'$	*2*				$\frac{Q''N''}{Q'N'} =$
$\Delta Q'''N'''N''$	*3*				$\frac{Q'''N'''}{Q''N''} =$
$\Delta Q''''N''''N'''$	*4*				$\frac{Q''''N''''}{Q'''N'''} =$

6. Record the successive ratios in the appropriate column of your table. Use an appropriate technology to generate a scatterplot of Leg Length vs. Dilation Number. Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

7. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

8. Use the parent function from Question 7 to determine a function rule to describe the relationship between dilation number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

9. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

10. What scale factor could be used to generate the second dilation from the original triangle without generating the first dilation?

11. What scale factor could be used to generate the third dilation from the original triangle without generating the first two dilations?

12. How could you predict the scale factor in terms of the dilation number?

13. What scale factor would be used to generate the 9th dilation in the sequence? What would the leg length of this triangle be? Explain how you determined your answer.

14. Which dilation will be the first one to have a leg length of at least 2.5 meters? Explain how you determined your answer.

A Golden Idea: Intentional Use of Data

TEKS		
Question(s) to Pose to Students — **Math**		
Question(s) to Pose to Students — **Tech**		
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement-Based Data Collection	
Bridge to the Classroom		