Leader Notes: The Doomsday Model

Elaborate

Purpose:

Use a problem context as a catalyst to generate a comparison of the strengths and weaknesses of different technologies. Generate a list of attributes to guide judicious use of technology.

Descriptor:

Participants will use a rational function model for population growth popularly known as the “Doomsday Model,” published by three scientists from the University of Illinois in 1960. Participants will obtain actual population data and verify the accuracy of the model using an appropriate technology, then communicate their findings. Participants will revise the model to better fit their data set, if necessary.

Participants will be asked to identify the strengths and weaknesses of using different types of technology. They will generate a list of attributes that can be used to guide judicious use of technology in their classrooms.

Duration:

2 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B)	Extend parent functions with parameters such as a in $f(x)=\frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.
2A.10(B)	analyze various representations of rational functions with respect to problem situations;
2A.10 (C)	determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;
2A.10 (D)	determine the solutions of rational equations using graphs, tables, and algebraic methods;
2A.10 (E)	determine solutions of rational inequalities using graphs and tables;
2A.10 (F)	analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- Internet access
- Graphing Calculator
- TI-Interactive
- Spreadsheet
- Graph linking capability, such as TI-Connect or Casio Program-Link

Materials:

Advanced Preparation: Transparencies

Presenter Materials: projector (computer or overhead) for graphing calculator

Per group: Internet access, sentence strips

Per participant: graphing calculator, activity sheets

Leader Notes:

In this phase of the professional development, participants will solve a problem by gathering data from the Internet, using their choice of technology to analyze the data, and be able to justify their choices. This activity will frame a discussion in which participants will be asked to identify the strengths and weaknesses of using different types of technology. They will generate a list of attributes that can be used to guide judicious use of technology in their classrooms.

Posing the Problem:
In 1960, Heinz von Foerster, Patricia Mora, and Larry Amiot, three scientists from the University of Illinois, published "Doomsday: Friday, 13 November, AD 2026" in the journal *Science*. In their paper, they considered the past population growth of the world and the current state (as of 1960) of the world's resources and their ability to sustain a certain population. They developed a model to describe population growth. A simplified variation of this model, where *t* represents the year and *P* represents the world population in billions, is:

$$P = \frac{195}{2026 - t}$$

They used this rational function to decide when the world's population would reach an unsustainable level and called this date "doomsday."

Use the Internet to obtain world population data since 1960. How well did the Doomsday Model describe the world's population growth between 1960 and 2000? How well does the model describe the world's population today? Based on the population data you found, how would you revise the model? When does this model predict "doomsday" will occur?

Share your results and your revised model with the group.

Facilitation Questions

- What kind of function is the Doomsday Model? What are its attributes?
  Answers may vary.
- What kind of function appears to model the actual population data? How do you know?
  Answers may vary.
- Which representation of the data would be most helpful?
  Answers may vary. Some may feel a scatterplot would be more helpful, and others would prefer a tabular or symbolic approach.
- Which technology would enable you to build this representation the most efficiently?
  Answers may vary, depending on the comfort level and experience of the participants with Excel, TI-Interactive, or the graphing calculator.

Leader Note: there are many possible solutions to this problem. In this phase of the professional development, it is more important to probe participants' reasoning for making their choices of technology. Participants' reasoning will help them build a framework for choosing the most appropriate technology in their day-to-day classroom instruction at the end of this phase of the institute.

One possible solution:
Participants can obtain world population data from the United Nations, the Central Intelligence Agency's World Factbook, or the U.S. Census Bureau. They can also obtain the data via online almanacs such as www.infoplease.com.

According to the United States Census Bureau:

Address http://www.census.gov/ipc/www/worldpop.html

U.S. Census Bureau

Total Midyear Population for the World: 1950-2050

Year	Population	Average annual growth rate (%)	Average annual population change
1950	2,556,517,137	1.47	37,798,160
1951	2,594,315,297	1.61	42,072,962
1952	2,636,388,259	1.71	45,350,197
1953	2,681,738,456	1.77	47,979,452
1954	2,729,717,908	1.87	51,465,740
1955	2,781,183,648	1.89	52,974,870
1956	2,834,158,518	1.95	55,842,882
1957	2,890,001,400	1.94	56,522,767
1958	2,946,524,167	1.76	52,351,768
1959	2,998,875,935	1.39	42,090,531
1960	3,040,966,466	1.33	40,782,196
1961	3,081,748,662	1.80	55,995,030
1962	3,137,743,692	2.19	69,519,033
1963	3,207,262,725	2.19	71,119,386
1964	3,278,382,111	2.08	68,979,816
1965	3,347,361,927	2.07	70,182,601
1966	3,417,544,528	2.02	69,689,877
1967	3,487,234,405	2.04	71,794,577
1968	3,559,028,982	2.07	74,579,864
1969	3,633,608,846	2.05	75,142,514

According to www.infoplease.com:

Year	*Total world population (mid-year figures)*	*Ten-year growth rate (%)*
1950	*2,556,000,053*	*18.9%*
1960	*3,039,451,023*	*22.0*
1970	*3,706,618,163*	*20.2*
1980	*4,453,831,714*	*18.5*
1990	*5,278,639,789*	*15.2*
2000	*6,082,966,429*	*12.6*

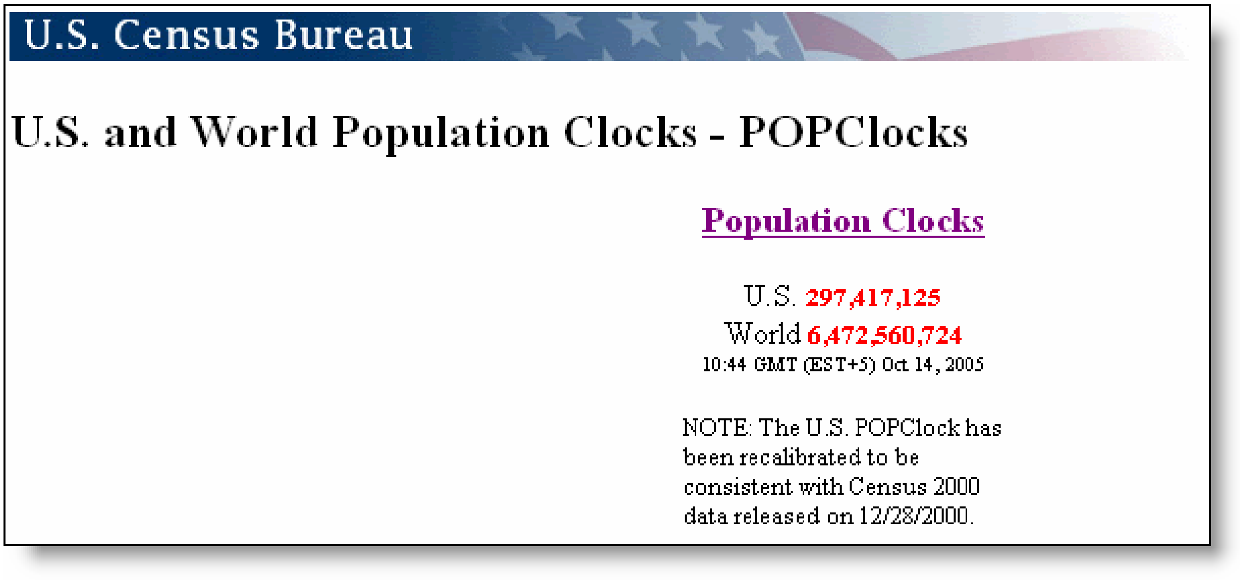
Use the function editor of a graphing calculator to build a table of the Doomsday Model function:

X	Y_1	
1985	4.7561	
1990	5.4167	
1995	6.2903	
2000	7.5	
2005	9.2857	
2010	12.188	
2015	17.727	

X=2000

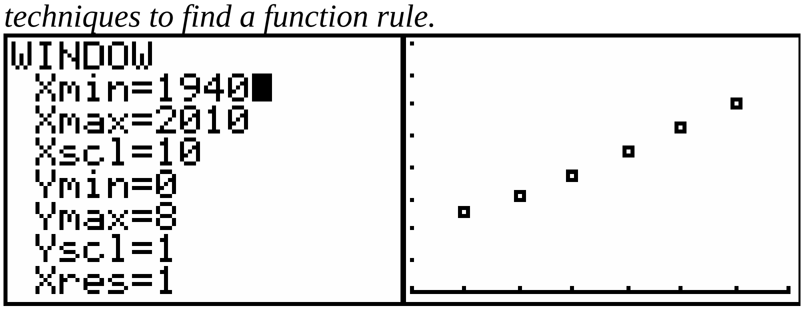
For the year 2000, the model predicted a population of about 7.5 billion people, and the actual population was about 6.08 billion, so the model actually overestimated the world's population.

According to the U.S. Census Bureau, www.census.gov,



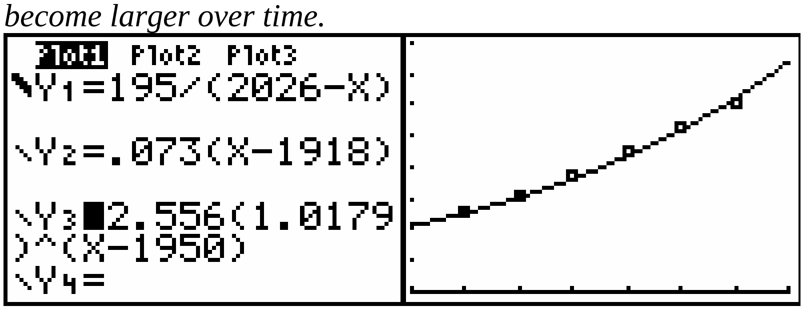
For today (actual year might vary, sample data shown for 2005), the model predicted a population of about 9.29 billion people, but the actual population estimate is only about 6.47 billion. Again, the model has overestimated the world's population.

To revise the model, use actual population data. Generate a scatterplot, then use curve-fitting techniques to find a function rule.

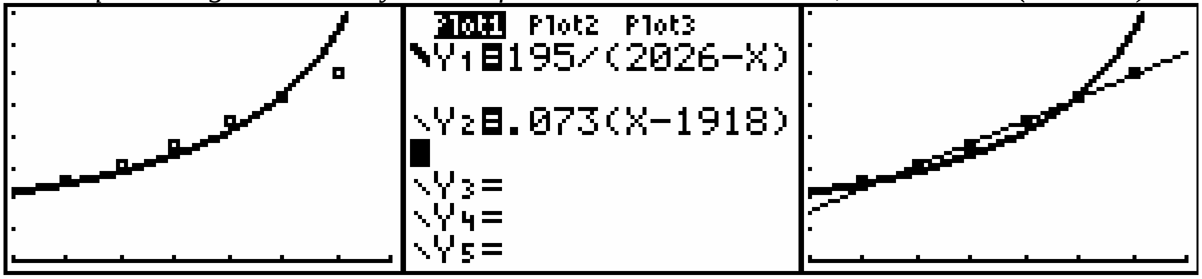


The Doomsday Model fits the data well until about 1990. However, the 2000 data point is well below the Doomsday Model curve (bold).

Possible Revised Models Include:
Population growth tends to be exponential. Try using transformations to generate an exponential function that will model the data. If participants use an exponential function, then there is no "doomsday" asymptote, but the population will continue to increase and the increase will become larger over time.



Visual inspection of the data reveals that the data appear to be linear. Rounded values of the least-squares regression line yield the function P = 0.073x – 140, or P = 0.073(x – 1918).



In factored form, the linear function has an x-intercept of 1918, meaning that in the year 1918, the world population was 0. Obviously, the model is not valid for years prior to 1950. In terms of determining the year of "doomsday," a linear function has no asymptotes. According to a linear model, the population can increase infinitely at a constant rate.

Debriefing the Activity:

1. *Upon completion of the technology-based activity, prompt participants to work in pairs to brainstorm the role(s) technology played in this activity.*

2. *Repost the Venn diagram summaries from the Engage phase.*

3. *Prompt participants to collect the "green sheets" from each Explore/Explain phase, the summaries about the intentional use of data that followed each Explore/Explain phase.*

4. *Display the* ***Transparency: Teaching Strategies*** *and prompt participants to reflect on the following question, "How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"*

Facilitation Questions

- How have the experiences in this professional development promoted careful decision-making about the appropriate use of technology?
  Answers may vary.
  E/E 1 example: technology can make complex problems accessible to all students
  E/E 2 example: comparing graphing calculator and spreadsheet to make scatterplots and generate function rules
  E/E 3 example: technology expands possible sources of data that can be used to explore functional relationships
  Elaborate example: there are multiple sources of Internet data, so the source of the data must be carefully considered
- How was technology used as a tool for the teaching and the learning of the TEKS?
  Answers may vary.
  E/E 1 example: use of calculator to solve a problem with complicated arithmetic
  E/E 2 example: the use of Geometer's Sketchpad to collect data
  E/E 3 example: use of light probe and CBL2 to collect data
  Elaborate example: use of the Internet to collect data to test and verify a model

- When was technology use promoted? Why?
  Answers may vary.
  E/E 1 example: participants are prompted to use a graphing calculator to generate a scatterplot and function rule
  E/E 2 example: participants are prompted to use technology to generate function rules, but are not told which technology to use
- When was technology use restricted? Why?
  Answers may vary. Overall, the use of technology was not overtly restricted in the TMT3 Algebra 2 module. However, NCTM suggests in their 2005 Yearbook, Technology-Supported Mathematics Learning Environments, that restricting the use of technology is an appropriate way to encourage learners to more judiciously choose which technologies to use in problem-solving and when to use them.
- How did the technology support anticipatory, or "what if…", thinking about "algebraic insight"?
  Answers may vary. Sample answers might include:
  Technology empowers students to quickly use transformations for curve-fitting to a set of data, building algebraic insight into functional relationships.
  Technology makes complex data sets and data collections accessible to all students.

5. *Post **Transparency 1: Looks Like – Sounds Like**. Prompt the participants to respond to the following statement and question: "A successful teacher is one who uses technology judiciously. What does this ideal teacher look like and sound like?" as described on Transparency 1: Looks Like – Sounds Like. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes as needed to facilitate the recording process.*

6. *Post **Transparency 2: Looks Like – Sounds Like**. Prompt the participants to respond to the following statement and question: "A successful student is one who uses technology judiciously. What does this ideal student look like and sound like?" as described on Transparency 2: Looks Like – Sounds Like. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes as needed to facilitate the recording process.*

7. *Direct the participants to work in small groups to brainstorm categories for classifying the "looks like" and "sounds like" responses.*

Facilitation Questions

- Do any of these responses require the teacher or the student to make decisions about technology use? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect decision making about how to best integrate technology? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect decision making about when to use or when not to use technology? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect the need for thinking about how the technology provides "algebraic insight"? Is this important? Should we add some responses?
  Answers may vary.

8. *As a whole group, debrief the categories created by small groups. Reorganize the sentence strips into broad categories. As a whole group, create titles for each of these categories. Record each title on a separate sheet of chart paper. Post the chart paper and reorganize the related sentence strips as shown below. Enlist participants to help with this process.*

Sample Category: Student Choice

The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide.

The student chooses to use a scatterplot instead of a table to represent her data.

9. *Prompt the participants to consider adding additional statements to any of the categories listed above that are not already posted. Reorganize "looks like, sounds like" sentence strips as needed.*

10. *Distribute to each group sentence strips that are a different color than the previously used sentence strips. Prompt each group to generate two classroom suggestions for each* ***category****. Examples may include "Students monitor their own use and misuse of technology," "Include examples that require technology use," or "Do not allow students to use technology until after predictions are made and justified."*

11. Prompt participants to post their sentence strips as shown below.

Sample
Category:
Student Choice

The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide.

The teacher provides a card whose front and back sides are two different colors, one color corresponding to calculator, one to computer. Students can display their choice of technology by placing the card with one color face up.

The teacher and students brainstorm a "pros and cons" chart to develop for the computer and the graphing calculator and then prompts students to select a tool.

12. Ask the participants to summarize any trends or patterns observed in the classroom suggestions.

13. Read the statement by Ball and Stacey found on ***Transparency: Student Research*** *as a closing thought to this phase of the professional development.*

Facilitation Question

- What is the value of this statement?
  Answers may vary. It is encouraging to read that technology use is teachable. It makes me consider how I might better meet the needs of the student who doesn't struggle with the math yet struggles with the technology.

Transparency: Teaching Strategies

"How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"

Judicious users of technology:

a. Promote careful decision-making about the appropriate use of technology.

b. Integrate technology whenever relevant to the mathematical learning goals.

c. Promote and restricts the use of technology when appropriate for promoting mathematical learning

d. Promote anticipatory thinking about "statistical insight," "algebraic insight," or "geometric insight."

Transparency 1: Looks Like – Sounds Like

A successful **teacher** is one who uses technology judiciously.

What does this ideal **teacher** look like and sound like in this activity?

Looks like…	Sounds like…

Transparency 2: Looks Like – Sounds Like

A successful **student** is one who uses technology judiciously.

What does this ideal **student** look like and sound like during the completion of this activity?

Looks like…	Sounds like…

Transparency: Student Research

Research by Pierce (2002) indicates that some students are always judicious users and others persist with passive or random, unthinking use. However, she found that a large, middle group can be helped to learn to work judiciously.

Ball & Stacey, 2005, p. 5

Ball, L., & Stacey, K. (2005). Teaching strategies for developing judicious technology use. In Masalski, W. J., & Elliott, P. C. (Eds.), *Technology-supported mathematics learning environments, sixty-seventh yearbook*, pp. 3-16. Reston, VA: National Council of Teachers of Mathematics.

The Doomsday Model

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